British Thornton 10″ slide rules

instructions for use

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Preface

This slide rule has been manufactured after a comprehensive study of the functional requirements involved and forms part of a new range designed for maximum efficiency. An outstanding design feature of this range is the introduction of end caps for holding the two stocks rigidly in relation to each other. These end caps are a major development in slide rule manufacturing technique. In addition to their primary function they enable the user to hold the slide rule by them and the recommended method of use is as follows:

(a) Hold the slide rule by the end caps

(b) When the slide is virtually fully contained in the two stocks manipulate the slide by the index fingers

(c) When the slide is extended to one end hold the rule by the end cap at the opposite end and manipulate the slide with the free hand

In this way lateral pressure across the width of the slide rule is avoided and highest practicable accuracy standards maintained

If treated with reasonable care and attention your slide rule will give you many years of good service

This instruction booklet deals with the use of the slide rule in separate stages for each major purpose and covers the main uses comprehensively. Certain portions may not apply to your particular slide rule as you may not have the scales involved —or if you have the scales they may not be relevant to the calculations in which you are interested. Thus if your slide rule has trigonometrical scales and you are not concerned with trigonometrical calculations then the stages dealing with these scales can be omitted

The purpose of having an instruction booklet covering the main uses is to help readers to see more readily the very wide variety of calculations where the slide rule can be of valuable assistance

The instructions cover all models in the British Thornton 25 cms (10 inch) range and reference to scale subdivisions are based on this range

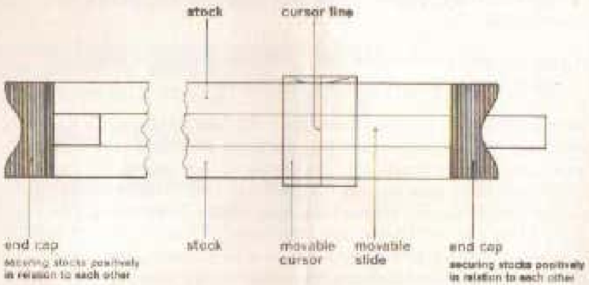
To the beginner

1 Introduction

It is easy to use a Slide Rule even though it may take practice to become really familiar with it. In using the various scales you will find it helpful to work out a simple problem which you can check mentally before going on to more complicated calculations. In this way confidence and a proper understanding of the scales is quickly built up, together with an appreciation of the very great use which can be made of the slide rule

Do not try to use the more advanced scales before you understand the basic scales and make a practice of rough checking your answer mentally—ask yourself 'Does it look right?'—and you will soon join the widening circle of slide rule initiates

2 Parts of the slide rule
To ensure that we understand the terminology here are the main parts of the slide rule



3 Significant figures
A slide rule can be regarded normally as giving the answer to a calculation to an accuracy of three significant figures (although sometimes a fourth significant figure can be read off). Significant figures do not have anything to do with the decimal point and must not be confused with it. If we take 276 as an illustration of three significant figures then 27,600; 276; 27.6; 0.276; 0.00276 are all examples of these three significant figures. Similarly with 408 as our three significant figures examples are 40,800; 4.08; 0.0408. Thus the number of "0's" to the left of the first significant figure or to the right of the third significant figure do not affect the significant figures themselves

4 Decimal point
Now a word about the position of the decimal point. Usually you know the approximate value of your answer and therefore the position of the decimal point — if there is any doubt then do a rough calculation and decide the position of it by estimation

5 C and D scales
Let us ignore all scales except the two identified by the letters C and D and which are located on the slide and stock respectively
These two scales are the most frequently used on a slide rule and are the basic scales normally used for multiplication, division, ratios etc

When we inspect these C and D scales closely we note that they are numbered from left to right viz., 1, 11, 12 and so on 2, 3, 4, 5 to 10. You will probably find it easier to imagine the numbering as 100, 110, 120 . . . 200, 300, 400, 500 . . . to 1,000 as this will help you in reading and setting the first three significant figures of numbers. The following illustration shows the settings for various significant figure values:

fig1

Notice how values between adjacent lines sub-dividing the scale change as we move along the scale. At the beginning the sub-divisions run 1, 101, 102, 103, 104, 105 etc up to 110 —thus giving a change of 1 in the third significant figure—and continue in this way up to the 2 position. The scale then runs 2, 202, 204 etc., thus giving a change of 2 in the third significant figure and further along the scale the change in third significant figure is 5. These changes in the values of sub-divisions on the various scales of the slide rule must be constantly observed

The C and D scales are logarithmic scales numbered naturally. This accounts for the fact that the distance between 1 and 2 is greater than the distance between 2 and 3 which in turn is greater than the distance between 3 and 4 and so on to 10. But the important thing to realise is that these logarithmic scales are of uniform proportional accuracy. A displacement of the C scale in relation to the D scale gives a fixed proportional relationship for all points in alignment. This can be seen by moving the slide to the right so that 1 on C scale is over 2 on D scale and observing that opposite 2 on C scale we have 4 on D scale opposite 3 on C we have 6 on D etc., thereby giving a portion of the two times table. Similarly by setting 1 on C scale over 3 on D scale we have a portion of the three times table

Now we are ready to commence using our slide rule

Instructions for use

Multiplication and division

DIVISION

This is carried out by subtracting one logarithmic length (the divisor) from another (the dividend).

Example 1

Evaluate $\frac{84}{24}$

Set the cursor at 84 on the D scale (referred as D_{84}) and move the slide so that 24 on the C scale (referred to as C_{24}) aligns with the cursor line. At 1 on C scale (referred to as C_1) read 35 on the D scale *ie* the significant figures of the answer

Decimal point considered = 3.5

Example 2

Evaluate $\frac{30.6}{68}$

Set the cursor at 306 on the D scale and move the slide so that 68 on the C scale aligns with the cursor line. At 10 on C scale read 45 on the D scale, the significant figures of the answer

Decimal point considered = .45

The two examples only differ in the respect that in example 1 the answer aligns with C_1 on the D scale and in example 2 the result is found at C_{10}

MULTIPLICATION

This is carried out by adding together the logarithmic lengths which correspond to the numbers

Example 3 Evaluate 2.6×3.5
Move the slide bringing C_1 to D_{26}, set the cursor at C_{35} and read on the D scale 91, the significant figures of 2.6×3.5.
Decimal point considered $2.6 \times 3.5 = 9.1$

Example 4 Evaluate 3.25×4.4

Cursor to D_{325}, C_{10} to cursor, cursor to C_{44} and read 143 on the D scale the significant figure value of 3.25×4.4, decimal point considered 14.3
Again it will be noted that the only difference between examples 3 and 4 is in respect of applying either C_1 or C_{10}, the choice being such as to bring the second factor within the D scale range

COMPOUND MULTIPLICATION AND DIVISION

The same principle applies of adding and subtracting logarithmic lengths corresponding to the numbers

The general rule is

FIRST numerator value is set on the D scale

ALL OTHER numerator and denominator values are set on the slide (thus adding or subtracting them from the logarithmic length of the first numerator value)

ANSWER is read on the D scale

Movement of the CURSOR carries out MULTIPLICATION

Movement of the SLIDE carries out DIVISION
and these operations must take place ALTERNATELY

If the CURSOR is moved last the result is read at the cursor on D scale

If the SLIDE is moved last the result is read on the D scale against the C_1 or C_{10} line

Example 5 Find the value of:

$$\frac{161 \times 923 \times 152}{258 \times 172}$$

Instruction	Stage	Significant figures of result on D scale at: C_1	C_{10}	Cursor
(*i*) Set cursor at D_{161}	161			
(*ii*) Move slide bringing C_{258} to the cursor	$\frac{161}{258}$		624	
(*iii*) Cursor to C_{923}	$\frac{161}{258} \times 923$			576
(*iv*) Move slide bringing C_{172} to the cursor	$\frac{161 \times 923}{258 \times 172}$	335		
(*v*) Cursor to C_{152}	$\frac{161 \times 923 \times 152}{258 \times 172}$			509

decimal point considered 509.0

In the next example, if we take the factors in the order in which they occur, alternating from numerator to denominator etc., what is known as an 'end switch' occurs as one of the factors on the slide to which it is desired to move the cursor extends beyond the end of the stock of the rule. The method involved for end switching is simply as follows:

When the slide protrudes to the left of the stock move the cursor to C_{10} and then 'end switch' the slide by bringing C_1 to the cursor. If the slide protrudes to the right of the stock move the cursor to C_1 and then 'end switch' the slide by bringing C_{10} to the cursor

'End switching' does not affect the accuracy of the answer as it is simply the equivalent of multiplying or dividing by 1 or 10 and the significant figures remain the same

Example 6 Find the value of:

$$\frac{.0535\times 741.0\times 4.87}{.1925\times .0524}$$

Instruction	Stage	Significant figures of result on D scale at: C_1	C_{10}	Cursor
(*i*) Set cursor at D_{535}	535			
(*ii*) Move slide C_{1925} to cursor	$\frac{535}{1925}$	278		
'End switch' (*iii*) Cursor to C_1				
(*iv*) Move slide C_{10} to cursor				
(*v*) Cursor to C_{741}	$\frac{535\times 741}{1925}$			206
(*vi*) Move slide C_{524} to cursor	$\frac{535\times 741}{1925\times 524}$		393	
(*vii*) Cursor to C_{487}	$\frac{535\times 741\times 487}{1925\times 524}$			1914

decimal point considered 19140.0

From the preceding examples it will be seen that for continued multiplication or continued division—using C and D scales only—we must divide or multiply by unity or 10 as required

Thus (using C and D scales only)

$$m\times n\times p$$

should be manipulated as:

$$m\div(1 \text{ or } 10)\times n\div(1 \text{ or } 10)\times p$$

Example 7 Evaluate $.0613\times 19.25\times .245\times 56.4$

(*i*) Cursor to D_{613} (*ii*) C_{10} to cursor (*iii*) Cursor to C_{1925} (*iv*) C_1 to cursor (*v*) Cursor to C_{245} (*vi*) C_{10} to cursor (*vii*) Cursor to C_{564} and on the D scale read 163 the significant figure value of the continuous multiplication; decimal point considered 16.3

Similarly $\frac{1}{q\times r\times s}$

should be worked as:

$$(1 \text{ or } 10)\div q\times(1 \text{ or } 10)\div r\times(1 \text{ or } 10)\div s$$

Example 8 Find the value of

$$\frac{1}{17.62\times .846\times 3.15}$$

(*i*) C_{1762} to D_1 (*ii*) Cursor to C_{10} (*iii*) C_{846} to cursor (*iv*) Cursor to C_{10} (*v*) C_{315} to cursor and read at C_1 on the D scale 213; decimal point considered .0213

Examples 7 and 8 have been worked in order to contrast with examples 9 and 10 using the available additional Reciprocal of C scale (on the slide) in conjunction with the C and D scales

CONTINUOUS MULTIPLICATION OR DIVISION

Using the Reciprocal of C scale (CI) in conjunction with the normal C and D scales. This reciprocal scale runs in the reverse direction to the C and D scales

The form: $m\times n\times p$
must be treated as

$$m\div\frac{1}{n}\times p$$

the $\frac{1}{n}$ being the n value on the *Reciprocal Scale* (CI)

Example 9 Evaluate $.0613\times 19.25\times .245\times 56.4$
treat as

$$.0613\div\frac{1}{19.25}\times .245\div\frac{1}{56.4}$$

(*i*) Cursor to D_{613} (*ii*) CI_{1925} to cursor (*iii*) Cursor to C_{245} (*iv*) CI_{564} to cursor and at C_1 on the D scale read 163; decimal point considered 16.3

Note only four settings required in place of seven in the corresponding example 7

Similarly: $\frac{1}{q\times r\times s}$
must be treated as

$$1\div q\times\frac{1}{r}\div s$$

Example 10 Find the value of

$$\frac{1}{17.62\times .846\times 3.15}$$

(*i*) C_{1762} to D_{10} (*ii*) Cursor to CI_{846} (*iii*) C_{315} to Cursor and read at C_1 on the D scale 213; decimal point considered .0213

Note the three settings in place of five required in example 8

Apart from the additional use of the Reciprocal Scale to obtain reciprocals by cursor projection from the C to the CI scale, other uses will occur to the user in relation to his particular computations

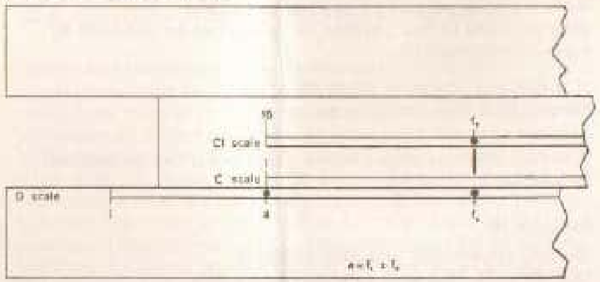


FIG 2

One relationship which merits special mention is:
Where $a=f_1\times f_2$ and 'a' is fixed, to determine any of the infinite pairs of values of f_1 and f_2 that will satisfy (see fig. 2)
Arrange slide so that C_1 (or C_{10}) is at value 'a' on the D scale
Using the Cursor for alignment from one factor on D scale, read other factor on the CI scale
Particular care must be taken, when using the Reciprocal of C scale, to keep in mind the reverse direction of ascending significant figures

USE OF CF AND DF SCALES

These scales are included on certain models of slide rules and occupy the position normally used for the A and B scales with DF on the stock and CF on the slide. These two scales are simply C and D scales displaced by the factor π, and they have particular advantages in multiplication, division, proportions, etc, as they give complete factor range in conjunction with C and D scales

This will be more easily appreciated by comparing the movements involved in a calculation using:

(a) C and D scales only

(b) CF and DF scales in conjunction with C and D

Complete Divisor Range

Example 11

Where $a+b+c+d+e=T$

express a, b, c, d, e as percentages of T given the following values:

$a=41.3$, $b=25.8$, $c=89.4$, $d=128$, $e=84.5$ and $T=369$

(a) *Using C and D scales only*

Move slide so that C_{369} aligns with D_1

Then for values 41.3, 84.5 and 89.4 move the cursor to
(*i*) C_{413} (*ii*) C_{845} (*iii*) C_{894}
and read the corresponding percentages on the D scale, *viz:*
11.2% 22.9% 24.2%
respectively

To obtain the remaining percentages for 128 and 25.8 it is necessary to move the slide so that C_{369} aligns with D_{10}, when they can be read by moving the cursor to

(*iv*) C_{128} (*v*) C_{258}
34.7% 7.0%

(b) *Using DF, CF, C and D scales*
Move the slide so that CF_{369} aligns with DF_{10}
With the slide in this position all values can be obtained by cursor projection, *viz:*

41.3, 84.5 and 89.4 from CF to DF scale
128 and 25.8 from C to D scale

The next example shows how a given factor can be applied to a series of numbers

Example 12

Multiply each of the following numbers:
12.7, 559, 173, 76.8, 24.6, 9.24 and 35.4 by .263
Using the combination DF and CF, C and D scales all the results can be obtained by a single displacement of the slide and relevant cursor projections *viz:*
Move slide so that CF_{10} aligns with DF_{263}
With the slide in this position all values can be obtained by cursor projection as follows:

From CF to DF scale—the products of
(*i*) 559 (*ii*) 76.8 (*iii*) 9.24 are read as
147 20.2 2.43 respectively

and from C to D scale—the products of
(*iv*) 12.7 (*v*) 173 (*vi*) 24.6 (*vii*) 35.4 read as
3.34 45.5 6.47 9.31

Two positions of the slide would have been necessary in the above example if only C and D scales were used

The next example deals with the familiar form

$$\frac{a \times b}{c}$$

Using C and D scales only, an 'end switch' is frequently necessary—as is the case in the example below—whereas with the combined displaced scales the 'end switch' can always be avoided by proper selection of the scale to which the initial factor 'a' is applied (*ie* a choice between D or DF)

The correct scale to choose is that which results in more than half the slide engaging in the stoc.. of the slide rule after the divisor has been set

Example 13

Evaluate $\dfrac{3.1 \times 8.15}{1.64}$

(a) *Using C and D scales only—and following normal practice*

(*i*) Cursor to D_{31} (*ii*) C_{164} to cursor (*iii*) End switch bringing cursor to C_1 and then C_{10} to cursor

(*iv*) Cursor to C_{815} (*v*) Read at the cursor 154 on D scale
After considering the decimal point

$$\frac{3.1 \times 8.15}{1.64} = 15.4$$

(b) *Using DF, CF, C and D scales*

(*i*) Cursor to D_{31} (*ii*) C_{164} to cursor (*iii*) Cursor to CF_{815}

(*iv*) Read at the cursor 154 on DF scale.
Decimal point considered=15.4

It must be borne in mind that a switch from the C, D pair of scales to the CF, DF pair of scales (and vice versa) is made when moving the cursor and not when moving the slide

Whilst the combination of the four scales minimises the need for 'end switching' it must not be concluded that it completely eliminates it in all cases of continuous compound calculations. With experience, by visualising the slide position before moving it, the user may often be able to select factors in an order, or choose scales to ensure that, after division, more than half of the slide is engaged with the stock. This will then mean that on either CF or C scales a complete significant figure range is in contact with either DF or D, thus enabling the cursor to be moved to the next factor without the need of an 'end switch'

Note—It will be appreciated that cursor projection from C and D scales to CF and DF is the equivalent of multiplication by π of the C or D scale value. Thus circumference from diameter of circle (and vice versa) can be obtained at a single setting of the cursor

Determination of Square Roots

It will be observed that Scale A of two $\frac{1}{2}$ unit sections is arranged in relation to the D scale (unit length) so that:

A_1 aligns with D_1
A_{100} aligns with D_{10}

Using the cursor for projection from D scale to A scale the following alignments can be observed

(*a*) for involution

D	1	2	3	4	5	etc	10
A	1	4	9	16	25	etc	100

ie 'Squares' of values on D are in alignment on A

(*b*) for evolution, a reverse process provides 'square roots' of the values on A in alignment with D scale

Since each section, *viz* 1 to 10 and 10 to 100, of the A scale provides a full cycle of significant figure range, in the case of 'square roots' the user has to decide which of the two sections is applicable to any particular evaluation

Consider numbers whose significant figure value is
2788
Such values may occur in various forms
as
.0002788, .002788, etc
or
278.8, 278800, etc

By way of illustration let us consider determination of the square roots of three of these say

$\sqrt{.0002788}$ $\sqrt{278.8}$ and $\sqrt{278800}$

Starting from the decimal point arrange bars over pairs of numbers as shown

(1) $\overline{.00}\ \overline{02}\ \overline{78}\ \overline{80}$

(1a) $\overline{2}\ \overline{78}\ \overline{.80}$

(2) $\overline{27}\ \overline{88}\ \overline{00}.$

(1) and (1a) are alike in the respect that the first significant figure 2 is alone under a bar, whereas in (2) the first two significant figures, that is 27, occur under the same bar.
In cases such as (1) or (1a) projection is from the first section of the A scale whilst case (2) calls for projection from the 2nd Section

Evaluate $\sqrt{\overline{.00}\ \overline{02}\ \overline{78}\ \overline{8}}$
0 1 6 7

Cursor to 2788 on 1st Section of A scale and read on D scale 167

Again consider the pairs, and with cipher or figure in the result for each pair, the significant figure 167, decimal point considered becomes .0167

Evaluate $\sqrt{\overline{2}\ \overline{78}\ .\ \overline{80}}$
1 6 . 7

Cursor to 2788 on 1st Section of A and read on D scale 167
This significant figure value 167, decimal point considered, becomes 16.7

Evaluate $\sqrt{\overline{27}\ \overline{88}\ \overline{00}.}$

Cursor to 2788 on the 2nd Section of A scale and read on D scale 528

This significant figure group 528, decimal point considered, becomes 528.0

After a little practice, the bars for pairing figures can be imagined and so the figure of a 'square root' together with its denomination, can readily be obtained by inspection

Determination of Cube Roots

Models which incorporate a Cube Root scale denoted by K furnish a direct means of obtaining cube roots by cursor projection. The scale comprises three repeats of a $\frac{1}{3}$ unit but care must be exercised in selection of the section to be used

Cube roots of numbers from 1 to 1000 are read off the D scale (C scale if the cube root scale is on the slide) by cursor projection from the K scale. For numbers above or below 1 to 1000 it is advisable to consider them in groups of three from the decimal point. Then the following basis can be applied:
One significant figure in excess of complete groups of three—use first section of K scale

Two significant figures in excess of complete groups of three—use second section of K scale

No significant figure in excess of complete groups of three—use third section of K scale

eg Evaluate 41780 and .04178

In both cases the two significant figures in excess of complete triads indicate the use of the second section of K scale from which we obtain 34.7 and 0.347 respectively as the cube roots

Determination of Logarithms

The Logarithm scale denoted by L is a uniform scale related to the C and D scales and provides logarithms to base 10

If we assign the definite values 1.0 to 10.0 to the significant figure scales C and D, then the length of L scale equals that of C or D from 1.0 to 10 and the extremes of the L scale are numbered 0 to 1.0 (Sub-divisions are in accord with decimal reading)

This combination functions as the equivalent of logarithm and antilog tables
To determine $\log_{10}5.2$

Use the cursor to project from $D_{5.2}$($C_{5.2}$ if L scale is on the slide) to the L scale and read $\log_{10}5.2 = .716$

The reverse process provides logarithm to number conversions

As with 'log' tables, only the 'mantissa' portion is obtainable from the rule and so in all cases, according to the position of the decimal point, the appropriate 'characteristic' must be applied

Orthodox Trigonometrical Scales

These scales are:
Sine scale, denoted by S, for the angle range 5.7 to 90°
Tangent scale, denoted by T, for the angle range 5.7 to 45°
Sine and Tangent scale, denoted by ST, for the angle range 0.57 to 5.7°

All three scales are decimally sub-divided, are related to the C and D scales and values are read off directly by cursor projection

eg Determine the value of Sine 20°
Set cursor to 20° on Sine scale and read on D scale at the cursor 342. Decimal point considered Sine 20° = .342

eg Determine Cos 16°
Since Cos 16° = Sine 74° treat as Sin 74°
Set cursor to 74° on Sine scale and read on D at the cursor 0.961 (the value of Sin 74° = Cos 16°)

eg Determine Tan 22°
Set cursor to 22° on Tan scale and read on D scale 0.404 = Tan 22°

Note: For tangents of angles between 45° and 90° use the formula

$$\text{Tan}\ \alpha = \frac{1}{\text{Tan}\ (90 - \alpha)} = \text{Cot}\ (90 - \alpha)$$

ie for Tan 75° set cursor to 15° on Tan scale and on DI read 3.73 = Tan 75° (if there is no DI scale then read on CI but see that C & D scales are aligned)

It will be appreciated that determination of the angle when the function value is given involves the inverse of the above process

eg To find the angle whose Sine is 0.41
Set cursor to D_{41} and read on S scale at the cursor 24.2°

The Sine and Tangent scale (ST) is used for both Sines and Tangents for the lower angle range below 5.7° and is the geometric mean of the two functions. In this respect it will be appreciated that Sin $\alpha \doteq$ Tan $\alpha \doteq \alpha$ in radians when α is small. Thus this scale may be used for converting degrees to radians and vice versa

Note: For small angles care is required in positioning the decimal point when reading answers on D scale, and in the selection of the appropriate angle scale when the function value is given. As a guide the following rule is useful:
If the angle is on the ST scale then .0 will precede the significant figures read off the D scale
If the function value on D scale is preceded by .0 then the angle is read on the ST scale

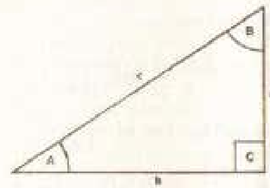
SOLUTION OF TRIANGLES

It is important to remember the Sine rule

$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$$

as these proportions can be usefully applied. It will be appreciated that if a given angle on the Sine scale is aligned with its respective given side on the C scale, then the other pairs of values are also in alignment. Thus if one of each of the other pairs is known the triangle can be solved by cursor projection between the Sine scale and the C scale

RIGHT ANGLED TRIANGLES



The Sine rule becomes $\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{1}$

Case 1

Given Angle A = 35.3° and side c = 533
To find sides a and b

Then $\frac{a}{\text{Sin 35.3}} = \frac{b}{\text{Sin (90—35.3)}} = \frac{533}{1}$

(a) Set C_{533} to D_{10}
(b) Cursor to 35.3 on Sine scale and read on C scale 308 = side a
(c) Cursor to (90—35.3) = 54.7 on Sine scale and read on C scale 435 = side b

Case 2

Given a = 207 and c = 305
To find Angle A and side b

Then $\frac{207}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{305}{1}$

(a) Set C_{305} to D_{10}
(b) Cursor to C_{207} and read on the Sine scale 42.75 = Angle A
(c) Cursor to (90—42.75) = 47.25 on Sine scale and read on C scale 224 = side b

Case 3

Given a = 133, b = 156
To find Angle A and side c

Then $\frac{133}{\text{Sin A}} = \frac{156}{\text{Sin B}} = \frac{c}{1}$

and Tan A = $\frac{133}{156}$

(a) Cursor to D_{133}
(b) C_{156} to cursor
(c) Cursor to C_{10} and read on Tangent scale 40.45 = Angle A
(d) Cursor to 40.45 on Sine scale
(e) C_{133} to cursor and at D_{10} read on C the value 205 = side c

Alternatively

(a) C_1 to D_{133}
(b) Cursor to 156 on Reciprocal of C scale (giving 133 × $\frac{1}{156} = \frac{133}{156}$) and read on the tangent scale 40.45 = Angle A
(c) Cursor to 40.45 on Sine scale and read on Reciprocal of C scale 205 = side c

Differential Trigonometrical Scales

The group of scales consists of the following:

Sine Differential scale (denoted by Sd) of $\frac{\alpha}{\text{Sin } \alpha}$ for Sine range 0 to 90°

Tangent Differential scale (denoted by Td) of $\frac{\alpha}{\text{Tan } \alpha}$ for Tangent range 0 to 60°

Inverse Sine Differential scale (denoted by ISd) of $\frac{x}{\text{Sin}^{-1}x}$ for inverse of above Sine range

Inverse Tangent Differential scale (denoted by ITd) of $\frac{x}{\text{Tan}^{-1}x}$ for inverse of above Tangent range

The above four scales are positioned on the slide and together take up the equivalent of one 'scale length'. They are used in conjunction with C and D scales and are very simple to manipulate

The principle is as follows:

Since $\text{Sd}_\alpha = \frac{\alpha}{\text{Sin } \alpha}$

Then $\frac{\alpha}{\text{Sd}_\alpha} = \frac{\alpha}{\frac{\alpha}{\text{Sin } \alpha}} = \alpha \times \frac{\text{Sin } \alpha}{\alpha} = \text{Sin } \alpha$

Thus by setting the α value on D scale and dividing by $\text{Sd}\alpha$ (*ie* the α value on the Sine Differential scale) the value of Sin α is read on D scale at C_1 (or C_{10})

eg To find Sin 43° Treat as $\frac{43}{\frac{43}{\text{Sin } 43}}$ *ie* $\frac{43}{\text{Sd}_{43}}$

Cursor to D_{43}
Bring 43° on the Sine Differential scale (*ie* Sd_{43}) to the cursor
At C_{10} read on D scale 0.682 = Sin 43°

eg To find Tan 36.5°
Cursor to D_{365}
Bring 36.5° on the Tangent Differential scale (*ie* $\text{Td}_{36.5}$) to the cursor
At C_{10} read on D scale 0.740 = Tan 36.5°

eg To find the angle whose Sine is 0.66 (*ie* the value of Sin^{-1} 0.66)

Treat as $\frac{0.66}{\frac{0.66}{\text{Sin}^{-1}0.66}}$ *ie* $\frac{0.66}{\text{ISd}_{0.66}}$

Cursor to D_{66}
Bring 0.66 on Inverse Sine Differential scale (*ie* $\text{ISd}_{0.66}$) to the cursor
At C_1 on D scale read 41.3 = Sin^{-1} 0.66

Similarly the angle whose Tangent is 0.9 is found to be 42° by using the Inverse Tangent Differential scale in conjunction with D scale

eg To find the value of 73 Sin 52°
Cursor to D_{73}
Bring Sd_{52} to the cursor
Cursor to C_{10} and read on D scale at the cursor 57.5

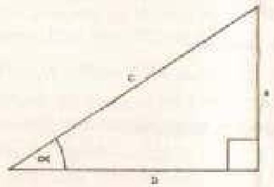
It will be appreciated that the Direct Sine and Tangent Differential scales provide the necessary *divisor correctives* which, when applied to angle readings on D scale, give the respective trigonometrical functions on D at C_1 or C_{10}. Similarly the Inverse Sine and Tangent Differential scales provide the *divisor correctives* which, when applied to function values on D scale, give the corresponding angles on D at C_1 or C_{10}

These Differential Trigonometrical scales give consistent maximum accuracy over the complete angle range and experience with them soon reveals their superiority compared with the orthodox Trigonometrical scales

The appearance of, for example, the Sine Differential Scale in the region of the 0° to 30° mark, in the respect that the distance is so comparatively short and the divisions so few, may suggest to the non-mathematical beginner that the accuracy is accordingly somewhat limited; but after a little practice with this scale, and thought regarding the nature of Sines, the user will correctly interpret the meaning of these small variations of the *divisor correctives* for the early angle range. Similar observations can be made with regard to the other scales. After comparison with tabulated and calculated results, users will soon realise that the highest significant figure accuracy possible by the C and D scales is consistently maintained by the Differential Scales over the complete angle range

On inspection of the Rule it will be observed that the Common Zero of the Direct Scales is at 'U' and coincides with the C scale reading 57.3, *ie* $\frac{180}{\pi}$ The Inverse Scales have their Common Zero at 'V' which corresponds to a C scale reading of 0.01745, *ie* $\frac{\pi}{180}$

RIGHT ANGLED TRIANGLES



eg Given a = 160 and b = 231

To find Angle α

$\frac{a}{b} = \text{Tan } \alpha$ *ie* $\alpha = \text{Tan}^{-1} \frac{a}{b} = \text{Tan}^{-1} \frac{160}{231}$

(a) Cursor to D_{160}

(b) Bring C_{231} to the cursor

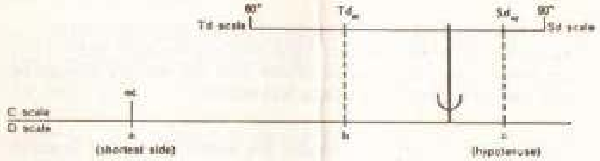
(c) Cursor to C_{10} and read on D scale 0.6925 = $\frac{a}{b}$

(d) $\text{ITd}_{0.6925}$ to cursor and read at C_1 on D scale 34.7° = α

When given one side and an angle (other than the right angle) the following relationship is useful.

$$\frac{a}{\alpha} = \frac{b}{Td_{\alpha}} = \frac{c}{Sd_{\alpha}}$$

This is expressed diagrammatically by



Thus the remaining two sides are obtainable at a single setting of the slide (except where an 'end switch' is involved when a second movement of the slide is necessary)

eg Given $c = 533$, $\alpha = 35.3°$

To find a and b

Cursor to D_{533} and bring $Sd_{35.3}$ in alignment.

Cursor to $Td_{35.3}$ and read on D in alignment $435 = b$

Cursor to $C_{35.3}$ and read on D in alignment $308 = a$

GAUGE MARKS V, U, m, s

These are conversion constants on the C scale for use as divisors as follows:

$$U = \frac{180}{\pi} = 57.2958 \text{ for Degrees to Radians}$$

$$V = \frac{\pi}{180} = 0.01746 \text{ for Radians to Degrees}$$

$$m = \frac{180 \times 60}{\pi} = 3437.75 \text{ for Minutes to Radians}$$

$$s = \frac{180 \times 60 \times 60}{\pi} = 206265.0 \text{ for Seconds to Radians}$$

Where $\alpha°$ is an angle expressed in Degrees
α' the angle expressed in Minutes
α'' the angle expressed in Seconds
ϕ the angle expressed in Radians

Then $\frac{\alpha°}{U} = \frac{\alpha'}{m} = \frac{\alpha''}{s} = \phi$

Note: For particular advantages of the Differential Trigonometrical scales in Tacheometric Surveying see Appendix

Log Log Scales

Note: The following instructions cover models with three direct and three reciprocal Log Log scales. Thus only certain portions of the instructions relate to models with two direct log log scales.

The Log Log scales and Reciprocal Log Log scales are arranged on the stock and are used for calculations involving the exponential form.

Any positive number N may be expressed as a particular power P of any positive base B thus $N = B^P$

$$\text{Hence } \operatorname{Log} N = P \operatorname{Log} B$$
$$\text{and } \operatorname{Log} \operatorname{Log} N = \operatorname{Log} P + \operatorname{Log} \operatorname{Log} B$$
$$\text{or } \operatorname{Log} \operatorname{Log} N - \operatorname{Log} \operatorname{Log} B = \operatorname{Log} P$$

ie the values B and N on the Log Log scale are separated by a distance representing Log P

eg Set cursor at 3 on LL3 scale and move slide bringing C_1 to the cursor

Observe by cursor projection that

2 on C scale aligns with 9 on LL3
3 on C scale aligns with 27 on LL3
4 on C scale aligns with 81 on LL3
5 on C scale aligns with 243 on LL3
thus evaluating 3^2, 3^3, 3^4, 3^5

Similarly, on models with Reciprocal Log Log scales, by setting the cursor at .3 on the LL03 scale and bringing C_1 to the cursor

2 on C scale aligns with .09 on LL03
3 on C scale aligns with .027 on LL03
4 on C scale aligns with .0081 on LL03
thus evaluating $(.3)^2$, $(.3)^3$, $(.3)^4$

eg Evaluate $N = 3.5^{2.66}$
Cursor to 3.5 on LL3 scale
Bring C_1 to the cursor
Move cursor to C_{266} and read on LL3 scale at the cursor
$28.0 =$ the value of N

Cursor projection from LL1 to LL2 or LL2 to LL3 scales effects the process of raising to the 10th power (or vice versa extracting the 10th root)

Thus $3.5^{.266} = 1.395$ the figure in alignment on the LL2 scale

When the base is less than unity the process is the same except that the Reciprocal Log Log scales are used

Thus $0.35^{2.66} = 0.0612$
and $0.35^{.266} = 0.7563$

eg Evaluate $(.35)^{2.66}$

(a) Cursor to .35 on LL03, C_1 to cursor and then move cursor to $C_{2.66}$. In alignment at cursor read .0612 on the LL03 scale

$(.35)^{2.66} = 0.0612$

(b) Read on the LL02 scale in the same alignment the value .7563
$(.35)^{.266} = .7563$

Note: In order to decide which scale provides the value, mental approximation is necessary. It may be said that most students are more at ease with rough approximations of powers of quantities *greater than unity* than with those of quantities *less than unity*

For example, it is easier to mentally appreciate that

$$3^{\frac{1}{2}} \text{ or } \sqrt{3} \simeq 1.7$$
$$3^3 = 27$$

than say

$$\sqrt{.4} \text{ or } .4^{\frac{1}{2}} \simeq .63$$
$$(.4)^3 = .064$$

Thus when dealing with the latter type as in the example it is useful to remember that at a particular setting on say the LL03 scale, the reciprocal of this value (greater than unity) appears in alignment on the related LL3 scale. The raising to the 'power' in question may then be observed on the greater than unity scales LL2 or LL3 and the final reading taken on the appropriate reciprocal log log scale

The student should also observe in respect of reciprocals that those obtained by projections from LL2 to LL02 and vice versa are more accurate than those obtained by C, D or CI scales; but for the range covered by the LL3 and LL03 scales the advantage is with the primary scales C and D

To solve for P when N and B are known, *ie* to determine the log of N to base B

Proceed as in the following example:
eg (Base > unity)
Solve $5.3^P = 92.0$

Set the cursor at 5.3 on LL3 scale and move the slide so that C_1 or C_{10} (in this case C_1) is at the cursor. Move the cursor to 92.0 on the LL3 scale and read the significant figures of P on the C scale at the cursor *viz* 271

$$\text{Then } 5.3^{2.71} = 92.0$$
or
$$\log_{5.3} 92 = 2.71$$

eg (Base less than unity)

$$.452^P = .764$$

(Obviously the value of P is less than unity)

Set the cursor at .452 on the LL02 scale. Align C_{10} at the cursor and move the cursor to .764 on LL02. On C scale at the cursor read 339 the significant figures of P . . . decimal point considered .339

To determine B when N and P are known

eg Determine B when $B^{2.14} = 40$
Set the cursor at 40 on LL3 and move the slide so that 2.14 on C is in alignment. Transfer the cursor to C_1 and note the readings on LL2 and LL3 namely 1.188 and 5.6 respectively

Mental approximation will immediately select 5.6 as the required value

$$ie\ 5.6^{2.14} = 40$$

and note that

$$1.188^{21.4} = 40$$

Note:

(i) That N and B are positions on the log log scales and their denominations must be respected, inasmuch as values for 1.45, 14.5, 145 and 1450 are distinct positions at different parts of the scales

(ii) That the significant figure value of P is employed on the C scale *ie* powers such as .25 and 2.5 are identical on that scale: denominations, supported by mental approximations, will dictate from which log log scale the N reading has to be taken

(iii) That 1 or 10 and P value on the C scale respectively align with B and N on the log log scales.

(iv) That where roots occur they must be re-expressed as 'powers'

eg $\sqrt[3]{\ }$ as the .333 power
$\sqrt[4]{\ }$ as the .25 power

(v) Normally a negative base cannot be raised to a power *eg* $(-5.94)^{3.41}$ cannot be evaluated: exceptions are where the 'power' is either an integer or the reciprocal of an odd integer

Where a 'power' and 'base' are such as to result in a value of N in excess of 2×10^4 as in the following

eg Evaluate $5.3^{7.8}$
Take the 'power' in parts as $7.8 = 4 + 3.8$
Evaluate 5.3^4 and $5.3^{3.8}$ and obtain

$$5.3^4 = 790 \qquad 5.3^{3.8} = 565$$
$$\text{Then } 5.3^{7.8} = 790 \times 565 = 446000$$

When N is greater than 2×10^4, to determine P for a given B proceed as follows

eg Determine P when $5.3^P = 446000$
Factorise 446000 as 1000×446
Then consider $P = q + r$ where $5.3^q = 1000$ and $5.3^r = 446$
Evaluate q and r and obtain $q = 4.14$ and $r = 3.66$

$$ie\ P = 4.14 + 3.66 = 7.8$$

Logarithms to the base e or evaluation of e^P

The log log scales are so positioned on the stock that projections therefrom on to the D scale furnish the significant figures of logarithms to the base e

Provisionally assign to the D scale the values 1.0 to 10 and remember that numbers in alignment on the log log scales bear the relationship

$$(LL2)^{10} = LL3 \qquad (LL02)^{10} = LL03$$

Place the cursor at say 1.326 on the LL2 scale and note that the D scale reading in alignment is 2.822.

Also observe reading on LL3 16.8
observe reading on LL02 .754
observe reading on LL03 .0595

It will be seen that

$$\log_e 1.326 = 0.2822 \text{ or } e^{.2822} = 1.326$$
$$\log_e 16.8 = 2.822 \text{ or } e^{2.822} = 16.8$$
$$\log_e 0.754 = -0.2822 \text{ or } e^{-.2822} = 0.754$$
$$\log_e 0.0595 = -2.822 \text{ or } e^{-2.822} = 0.0595$$

The alignment of values is useful when evaluating hyperbolic functions since:

$$\text{Sin h}x = \frac{e^x - e^{-x}}{2}$$

$$\text{Cos h}x = \frac{e^x + e^{-x}}{2}$$

$$\text{Tan h}x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\text{Sin h}x}{\text{Cos h}x}$$

In the case of $x = 2.822$

$$\text{Then Sin h}x = \frac{16.8 - 0.0595}{2} = 8.37$$

$$\text{Cos h}x = \frac{16.8 + 0.0595}{2} = 8.43$$

$$\text{Tan h}x = \frac{8.37}{8.43} = 0.993$$

Use of the L Constant

The L constant at 2.3026 on C scale is useful for converting logs to base e to logs to base 10 since

$$\text{Log}_{10} N = \frac{\text{Log}_e N}{2.3026} = \frac{\text{Log}_e N}{L}$$

The conversion is effected by bringing the L mark on C scale in alignment by cursor projection with the N value on the log log scale and reading the value of $\text{Log}_{10}N$ on D scale at C_1 (or C_{10})

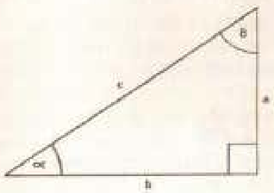
It will be realised that logarithms to any base can be obtained by making a mark on C scale in the position which aligns with the particular base on the log log scales (with C_1 and D_1 in alignment of course) and by using the position marked on the C scale as a divisor for that base

eg To find $\text{Log}_2 8$
(a) With C_1 and D_1 in alignment move cursor to 2.0 on log log scale and then make a pencil mark on C scale at the cursor position, namely 693
(b) Transfer cursor to 8 on log log scale
(c) Bring marked position at C_{693} to the cursor and read 3 on D scale at C_{10}
Then $\log_2 8 = 3$ (or $8 = 2^3$)

Vector Analysis Scales

Ps of $\sqrt{1 - s^2}$ and Pt of $\sqrt{1 + t^2}$

With the Ps scale of $\sqrt{1 - s^2}$ it is possible, by cursor projection to the D scale, to obtain Cos α when Sin α is known or vice versa



The difference between two squares is treated as follows

$$x = \sqrt{c^2 - a^2} = c\sqrt{1 - \left(\frac{a}{c}\right)^2}$$

Since $\frac{a}{c} = \sin\alpha = s$

Then $x = c\sqrt{1 - s^2}$

eg Evaluate $\sqrt{5.3^2 - 2.8^2}$

$$= 5.3\sqrt{1 - \left(\frac{2.8}{5.3}\right)^2}$$

(I) Cursor to D_{28}, C_{53} to cursor and at C_{10} on D read

$$0.528 = \frac{2.8}{5.3}$$

(II) Cursor to .528 on Ps scale, C_{10} to cursor, cursor to C_{53} and read 45 on D scale
Then $\sqrt{5.3^2 - 2.8^2} = 4.5$

Note: Where s is greater than 0.995 the form $\sqrt{2(1 - s)}$ may be used as a close approximation

The Pt scale of $\sqrt{1 + t^2}$ serves for determination of the square root of the sum of two squares as under:

$$h = \sqrt{a^2 + b^2} = b\sqrt{1 + \left(\frac{a}{b}\right)^2}$$

Since $\frac{a}{b} = \tan\alpha = t$

Then $h = b\sqrt{1 + t^2}$

eg Evaluate $\sqrt{3^2 + 4^2}$

$$= 4\sqrt{1 + \left(\frac{3}{4}\right)^2} = 4\sqrt{1 + (0.75)^2}$$

(*i*) Cursor to 0.75 on Pt scale
(*ii*) Bring C_1 to cursor
(*iii*) Cursor to C_4 and read on D scale the value of $h = 5$

Cursors

Single line cursors are supplied as standard since three line type cursors can be confusing. An exception to this however is the P221 Comprehensive model which has an additional line to the left of the centre line covering the A and B scales only and giving the $\pi/4$ constant in relation to the centre line

The left hand line may be used for calculations involving areas of circles where diameter is given and vice versa and the distance between the left hand line and the main cursor line corresponds to the interval 0.7854 to 1 on the A scale

By setting the main line to a diameter on D scale the corresponding area of the circle can be read on A scale at the left hand line since passing from D to A scale squares the diameter and reading at the left hand line is the equivalent of multiplying on A scale by $0.7854 = \frac{\pi}{4}$

The formula used is thus $\text{Area} = \frac{\pi}{4}d^2$

It will be appreciated that area to diameter conversions may be made in the opposite way
eg Given area of circle is 120 sq ins
Find the diameter

(a) Set left hand cursor line to 1.2 on A scale
(b) From the main cursor line read on D scale 1236. Then diameter of circle equals 12.36 inches

On the traditional type of three line cursor the right hand line covered horse power to kilowatt conversions, the distance between the main cursor line and the right hand line corresponding to the interval 0.746 to 1 on the A scale

eg Find the number of kilowatts in 150 hp
(a) Set right hand cursor line to 1.5 on A scale
(b) At the main cursor line read 112. Then number of kilowatts equals 112

Appendix—Tacheometric Surveying

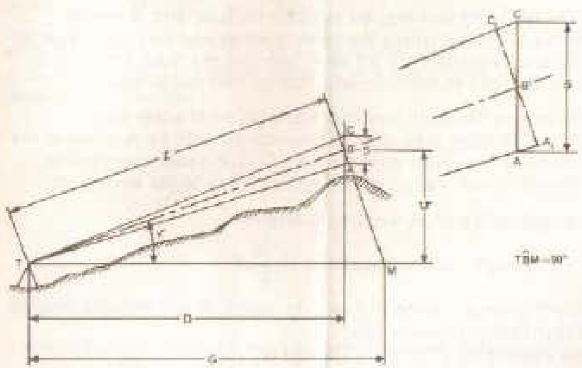
STADIA COMPUTATIONS

In the instructions for Differential Trigonometrical scales reference was made to the complete solution of the triangle given one side and one angle (other than the right angle)

Since the computations of heights and distances from stadia data call for the treatment of two overlapping right angle triangles, it will be seen from the following that a rule which incorporates the differential Sine and Tangent scales provides the means of effecting the evaluation of D, v and Z (see figure) when G and V° are known, by a *single setting of the slide* (occasional end switching excepted)

In the following it is assumed that the tacheometric instrument used is provided with an analatic lens, also that the measurement staff is held in a vertical position

Where V° = the inclination of the telescope
- S = the (vertical) staff reading = AC
- r = the 'corrected' (staff) reading = A_1C_1
- D = the horizontal distance
- v = the vertical distance
- Z = Distance on Collimation Line
- G = Uncorrected distance Z = k S
- k = Constant for the wires (usually 100)



eg

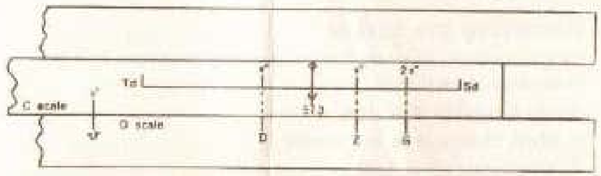
Determine D, v and Z, when S = 4.5 ft and V° = 20° and the constant for the stadia lines = 100
G = 100 × 4.5 = 450, V° = 20°, 2V° = 40°

Set the slide so that the 40° mark on the Sd scale aligns with 450 on the D scale

Then in alignment with:

20° on the Sd scale read the value of Z = 423 on the D scale
20° on the Td scale read the value of D = 397.4 on the D scale
20 on the C scale read the value of v = 144.6 on the D scale

That is, evaluate in accordance with the following diagram



In the case of a value of G which occurs towards the left of the D scale such that Sd_2v° when aligned with G results in Sd_v°, Td_v° or C_v° being to the left of D_1, then end switching must be effected by placing the cursor at C_{10} and then bringing C_1 to the cursor. This will enable the values of the remaining projections on the D scale to be made

The mathematical proof which substantiates the correctness of the foregoing method follows

From the figure and remembering that

$$\text{Sin } V^\circ \text{ Cos } V^\circ = \frac{\text{Sin } 2V^\circ}{2}$$

we get:

$$v = G \text{ Sin } V^\circ \text{ Cos } V^\circ = \text{Tan } V^\circ \, G \text{ Cos}^2 V^\circ = D \text{ Tan } V^\circ =$$

$$Z \text{ Sin } V^\circ = G \, \frac{\text{Sin } 2V^\circ}{2}$$

Then

$$\frac{V^\circ}{v} = \frac{V^\circ}{G \text{ Sin } V^\circ \text{ Cos } V^\circ} = \frac{V^\circ}{\text{Tan } V^\circ \, G \text{ Cos}^2 V^\circ}$$

$$= \frac{V^\circ}{D \text{ Tan } V^\circ}$$

$$= \frac{V^\circ}{Z \text{ Sin } V^\circ} = \frac{2V^\circ}{G \text{ Sin } 2V^\circ}$$

$$\frac{V^\circ}{v} = \frac{\dfrac{V^\circ}{\text{Tan } V^\circ}}{D} = \frac{\dfrac{V^\circ}{\text{Sin } V^\circ}}{Z} = \frac{\dfrac{2V^\circ}{\text{Sin } 2V^\circ}}{G}$$

ie

$$\frac{V^\circ}{v} = \frac{Td\,v^\circ}{D} = \frac{Sd\,v^\circ}{Z} = \frac{Sd_2v^\circ}{G} = \frac{Sd_2v^\circ}{ks}$$

The slide rule interpretation of this form is given in the previous figure

When the angle V° is small (say less than 3°), then within the accuracy limits of measuring 'S'

(i) $S \simeq r$ (ii) $Z \simeq 100r$

(iii) $D \simeq Z$ (iv) $v \simeq \dfrac{100r\,V^\circ}{57.3}$

eg Given V° = 1° 14′ (*ie* less than 3°), and r = 6.17
To determine D and 'v'
Express 1° 14′ as a degree and decimal = 1.233°

$$D \simeq 100 \times 6.17 = 617$$

To obtain 'v' evaluate $\dfrac{D\,V^\circ}{57.3} = \dfrac{617 \times 1.233}{57.3}$

Set cursor at D_{617}, U (at $C_{57.3}$) to cursor, cursor to 1.233
Read 'v' at cursor on D = 13.28 ft

Care and attention

Removing the cursor

This is sometimes desirable for cleaning purposes and the procedure is as follows:—

SINGLE SIDED CURSOR

1 Move slide to one end of rule
2 Centralise the cursor
3 Compress the rule across its width in the region of the cursor which can now be removed

DOUBLE SIDED CURSOR

1 Unscrew the four screws on one side of the cursor and note whether cursor spring is at top or bottom (preferably the reverse side plate should be unscrewed)
2 Remove cursor plate and clean cursor as necessary
3 Replace the cursor with the cursor spring in correct position. Insert the screws and tighten up sufficiently to hold the plate in position
4 Check that the black cursor line is square across the scales (adjusting if necessary) and then tighten up slightly to secure the plate

Note: The double sided cursor may also be cleaned by sliding a piece of tissue paper under the plates as an alternative to removal

Cleaning the slide rule

The slide rule may be cleaned simply by washing it in a lukewarm solution of soap and water

Major historical developments in the evolution of the slide rule

*1614 Invention of logarithms by John Napier, Baron of Merchiston, Scotland

*1617 Development of logarithms 'to base 10' by Henry Briggs, Professor of Mathematics, Oxford University

*1620 Interpretation of logarithmic scale form by Edmund Gunter, Professor of Astronomy, London

*1630 Invention of slide rule by the Reverend William Oughtred, London

*1657 Development of the moving slide/fixed stock principle by Seth Partridge, Surveyor and Mathematician

*1775 Development of the slide rule cursor by John Robertson of the Royal Academy

1815 Invention of the log log scale principle by P. M. Roget of France

*1900 Re-introduction of log log scales by Professor Perry, Royal College of Science, London

*1933 Differential trigonometrical and log log scales invented by Hubert Boardman, Radcliffe, Lancashire

*denotes British development

BRITISH THORNTON LTD
PO Box 3, Wythenshawe, Manchester 22

Booklet designed by
STEVENSON/WARD
Brunswick Hill, Macclesfield, Cheshire

Printed in England by
T JAMIESON & CO LTD
Salford, Lancs