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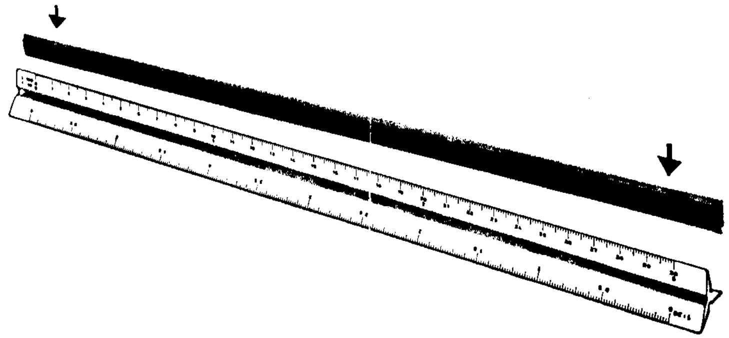

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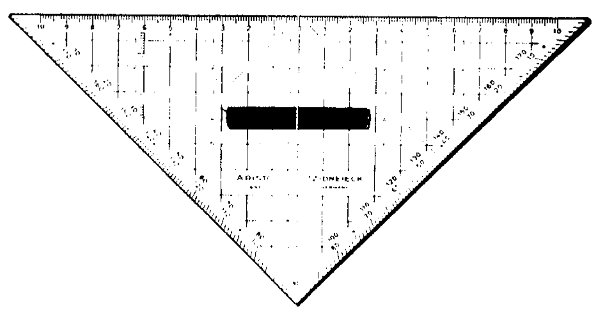

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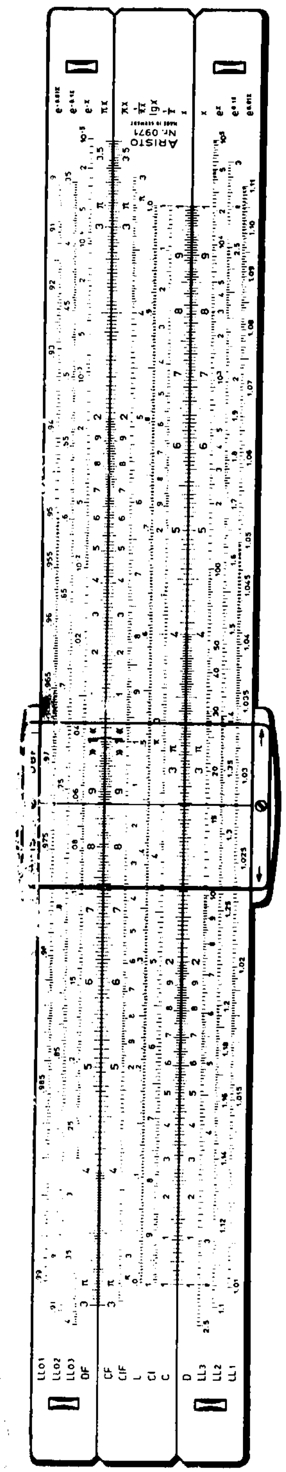

ARISTO

HYPERBOLOG
HYPERLOG

E

The slide rule ARISTO HyperboLog 0971

The ARISTO HyperboLog is a universal LogLog slide rule in which the system of graduations includes the scales for the hyperbolic functions sinh and tanh. It has been designed for the convenience of men who constantly use hyperbolic functions in their professional work. Computations in high frequency and telecommunication engineering, for example, can be performed with remarkable facility, convenience and speed.

The slide rule ARISTO HyperLog 0972

A double-face slide rule for mathematicians, physicists and communications engineers.

In addition to the hyperbolic function scales Sh1, Sh2 and Th of the ARISTO HyperboLog, this rule has a scale Ch for hyperbolic cosines and two hyperbolic function scales H1 and H2 $\left(\sqrt{1 + x^2}\right)$.

The trigonometrical function scales are supplemented by a Pythagoras scale P and the LogLog scale by the sections LL0 and LL00.

CONTENTS

1. General introduction 5
 1.1 Manipulation of the slide rule 5
 1.2 Personal identification tab 5
 1.3 Treatment of the ARISTO slide rule 6
 1.4 Removing the cursor at the slide rule 0971 6
 1.5 Adjustment of the cursor at the slide rule 0971 6
 1.6 Removing the cursor at the slide rule 0972 6
 1.7 Adjustment of the cursor at the slide rule 0972 7
 1.8 Working diagrams used in the solution of examples 7
2. Scale arrangement
 ARISTO HyperboLog 7
 ARISTO HyperLog 9
3. Reading the scales 12
4. Making approximations 13
5. The slide rule principle 13
6. Multiplication 14
7. Division 14
8. The folded scales CF and DF 15
 8.1 Tabulation without slide re-setting 15
 8.2 Direct reading of products and quotients involving π 16
9. Combined multiplication and division 16
10. Reciprocal scales CI and CIF 16
 10.1 Reciprocal scale DI 18
11. Proportions 18
12. Scale A, B and K 18
 12.1 Calculation with scales A and B 19
13. Pythagoras scales 19
 13.1 Scale P 19
 13.2 Scales H1 and H2 20
14. Trigonometrical functions 21
 14.1 Sine scale S 21
 14.2 Tangent scale T 22
15. Scale ST 22
 15.1 Small angles — large angles 23
 15.2 Conversion: circular ↔ radian measure 23
 15.3 The marks ϱ' and ϱ'' 24
16. Trigonometric solution of plane triangles 24
 16.1 Complex numbers 26
17. The LogLog scales 26
 17.1 Powers and roots with exponents 10 and 100 27
 17.2 Powers: $y = a^x$ 27
 17.3 Special cases of $y = a^x$ 29
 17.4 Powers: $y = e^x$ 32
 17.5 Roots: $a = \sqrt[x]{y}$ 32
 17.6 Logarithms 33
18. Further applications of the LogLog scales 35
 18.1 Solving proportions with the LogLog scales 35

19. The hyperbolic functions 37
19.1 Scales Sh1 and Sh2 37
19.2 Scale Ch 38
19.3 Scale Th 39
19.4 Fundamental hyperbolic formulae 39

20. Hyperbolic functions with complex argument 40
20.1 sinh (x + j y) 40
20.2 cosh (x + j y) 43
20.3 tanh (x + j y) 46

21. Trigonometric functions with complex argument 46

22. Conversion of examples of sections 20 and 21 46
22.1 ar sinh $r/\varphi = x + j y$ 47
22.2 ar cosh $r/\varphi = x + j y$ 47
22.3 ar tanh $r/\varphi = x + j y$ 48
22.4 arc sin $r/\varphi = x + j y$ 48
22.5 arc cos $r/\varphi = x + j y$ 48
22.6 arc tan $r/\varphi = x + j y$ 49

23. Examples of application in practice 49

24. The cursor and its marks 53
24.1 The mark 36 53
24.2 Circular areas, weights of bar steel 53
24.3 Marks kW and HP 54
24.4 The gauge marks 2π at the cursor L 0972 54

25. Scale of preferred numbers 1364 54
25.1 Construction of scale of preferred numbers, NZ 54
25.2 Objects of NZ scale 55
25.3 Logarithmic scales 55
25.4 Conversion factors for non-metric units 55

1. General introduction

In this instruction book information is given concerning the scales of the slide rule, their range and purpose. Calculations are explained, together with the interrelationships of the scales. To clarify principles involved, examples of use are given for each scale, and guidance in arranging the most important factors in complex formulae.

Expertise in slide rule manipulation comes with practice. Further exercises and detailed explanations are to be found in the textbooks recommended:

Stender and McKelvey: The Modern Slide Rule

Ellis, J. P.: The Theory and Operation of the Slide Rule

1.1 Manipulation of the slide rule

When using the slide rule it is best so to hold it that the light, falling on the cursor, does not throw a shadow of the cursor line. The most precise movement of the slide results from pressure and counter pressure. The projecting end of the slide should be held by the index finger and thumb, close to the body of the rule. Movement of the finger and simultaneous pressure against the rule body achieves the desirable smooth pulling and pushing action. The other hand holds the body of the rule by the upper body panel, so that the thumb can be used to press against the end of the slide.

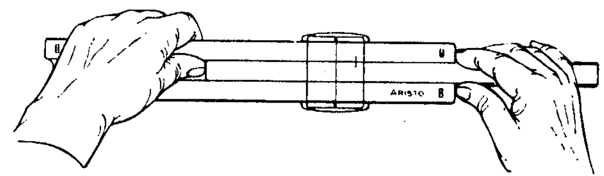

Fig. 1

Setting the cursor is possible, using either hand, but is more speedily and more accurately accomplished by using the thumb and index finger of both hands. By lightly pressing the bearing edge of the cursor, opposite the cursor spring, against the edge of the rule body, tilting the cursor is avoided and the cursor hairline is maintained perpendicular to the scales.

1.2 Personal identification tab

In the case of the slide rule, under the ARISTO scale of preferred numbers 1364, will be found a transparent insert, which can be used to identify ownership of the slide rule. The card contained can be removed, after bending the transparent flap upwards and the name of the owner of the rule can then be written on the card.

1.3 Treatment of the ARISTO slide rule

The slide rule is a valuable calculating aid and deserves careful treatment. Scale faces and cursor should be protected from dirt and scratches, so that reading accuracy may not suffer.

It is advisable to give the rule an occasional treatment with the special cleansing fluid, DEPAROL, followed by dry polishing. The use of chemical substances of any description should be avoided as they may damage the scales.

Protect the slide rule from plastics erasers and their abrasive dusts, which can cause damage to the ARISTOPAL rule faces. Do not place the rule on hot surfaces such as radiators, or expose it to full sunlight. Deformation is likely to occur at temperatures above 140° F (60° C). Rules so damaged will not be replaced free of charge.

1.4 Removing the cursor at the slide rule 0971

The cursor lines are so adjusted, with reference to the scale pattern, that it is possible, if convenient, to transfer work from one face of the rule to the other during the course of a calculation. The cursor can be removed for cleaning without upsetting this adjustment. On one face of the cursor, the glass is held by four screws. On the opposite face, the glass is retained by two press studs which enter the cursor bridge pieces. To remove the cursor, the cursor bridge is pressed downwards at the points marked by arrow heads, with the thumb nails, to release the press stud. The upper press stud is released by tilting the cursor glass upwards.

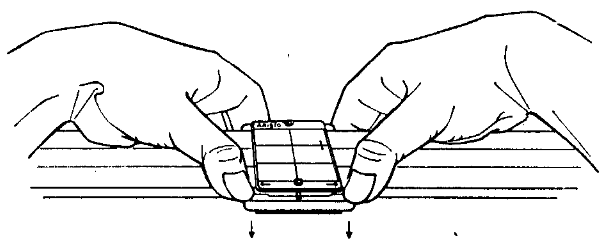

Fig. 2

1.5 Adjustment of the cursor at the slide rule 0971

It is occasionally necessary, as when fitting a replacement cursor, to adjust the position of the hairlines with reference to the scales. To do this, the rule is laid on the table with the face of the cursor bearing the four screws, uppermost. After loosening these four screws with a suitable screwdriver, the rule is turned over and the cursor hairline accurately adjusted to the index lines e^x and e^{-x} in the scales LL2 and LL02. The rule is then very carefully turned over again, without moving the cursor. Whilst holding the cursor firmly in place, the upper cursor glass is set to the index marks 1 or 1000 of the DI and K scales. This done, the four screws are firmly retightened.

1.6 Removing the cursor at the slide rule 0972

The cursor hairlines are so adjusted that transfer from one face of the rule to the other is possible at any stage in a calculation. The cursor can be removed, for cleaning, without disturbing this adjustment — provided that the screw on the cursor bridge piece is not lost.

To remove the cursor, hold firmly, with one hand, the screwed cursor bridge piece. The other cursor bridge piece — that without the screw — can then be released by a rotary movement of the screwed cleat and cursor glasses across the face of the rule, as shown in fig. 2a. Cursor glasses and bridge pieces can then be removed.

When replacing the cursor, take care to set it with the gauge marks kW and HP over scales A and B. The sprung cursor bridge piece should then be brought over the cursor glasses and the assembly closed by light pressure.

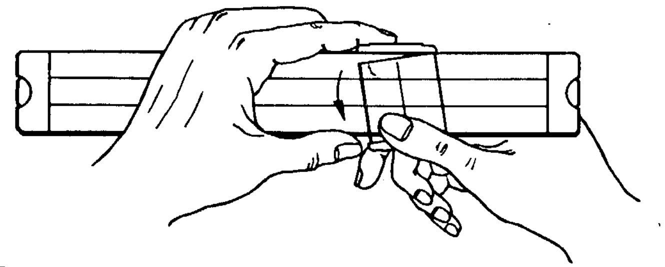

Fig. 2a

1.7 Adjustment of the cursor at the slide rule 0972

After loosening the adjustment screw of the cursor, the rule should be turned over so that the cursor hairline can be set to the auxiliary marks on the LL scales. Without moving the cursor, turn the rule over again and place it on the table. The now facing cursor hairline can be set to the right hand index marks of scales A and D. This done, the adjusting screw can be re-tightened.

1.8 Working diagrams used in the solution of examples

In what follows a method of representation will be used to show, in a form more easily followed than in the more usual slide rule diagrams, the process of solution and sequences of setting. The scales are represented by parallel lines, at the ends of which the scale identifications are given. The undermentioned symbols aid interpretation of the diagrams.

Initial setting

Each subsequent setting

Final result

Setting or reading an intermediate result

Reversing the rule

Arrowhead showing sequence and direction of movement

Cursor line shown by a perpendicular.

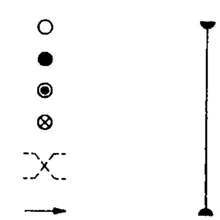

Fig. 3

2. Scale arrangement

THE SLIDE RULE ARISTO HYPERBOLOG 0971

The LogLog face

Scale	Description	Function	Location
LL01	LogLog scales, range 0.99 to 0.9	$e^{-0.01x}$	on rule body
LL02	0.91 to 0.35	$e^{-0.1x}$	on rule body
LL03	0.4 to 10^{-5}	e^{-x}	on rule body
DF	Folded scale	πx	on rule body
CF	Folded scale	πx	on the slide
CIF	Reciprocal scale of CF	$1/\pi x$	on the slide
L	Mantissa scale	lg x	on the slide
CI	Reciprocal scale of C	1/x	on the slide
C	Fundamental scale	x	on the slide
D	Fundamental scale	x	on rule body
LL3	LogLog scales, range 2.5 to 10^5	e^x	on rule body
LL2	1.1 to 3.0	$e^{0.1x}$	on rule body
LL1	1.01 to 1.11	$e^{0.01x}$	on rule body

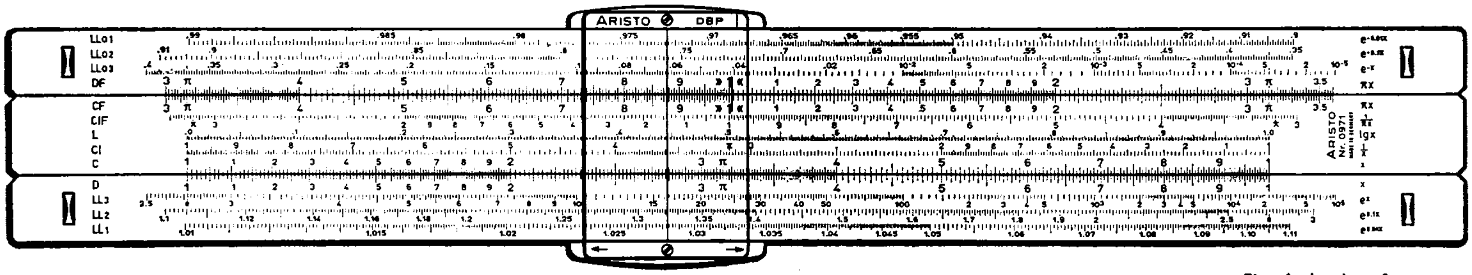

Fig. 4 LogLog face

The hyperbolic face

Scale	Description	Function	Location
Th	Scale of hyperbolic tangents for arguments from 0.1 to 3.0	∢ tanh	On rule body
K	Scale of cubes	x^3	On rule body
A	Scale of squares	x^2	On rule body
B	Scale of squares	x^2	On the slide
T	Scale of tangents and cotangents	∢ tan	On the slide
ST	Scale of small angles in radians	∢ arc	On the slide
S	Scale of sines and cosines	∢ sin	On the slide
C	Fundamental scale	x	On the slide
D	Fundamental scale	x	On rule body
DI	Reciprocal scale	1/x	On rule body
Sh2	Scale of hyperbolic sines for arguments from 0.85 to 3.0	∢ sinh	On rule body
Sh1	0.1 to 0.9	∢ sinh	On rule body

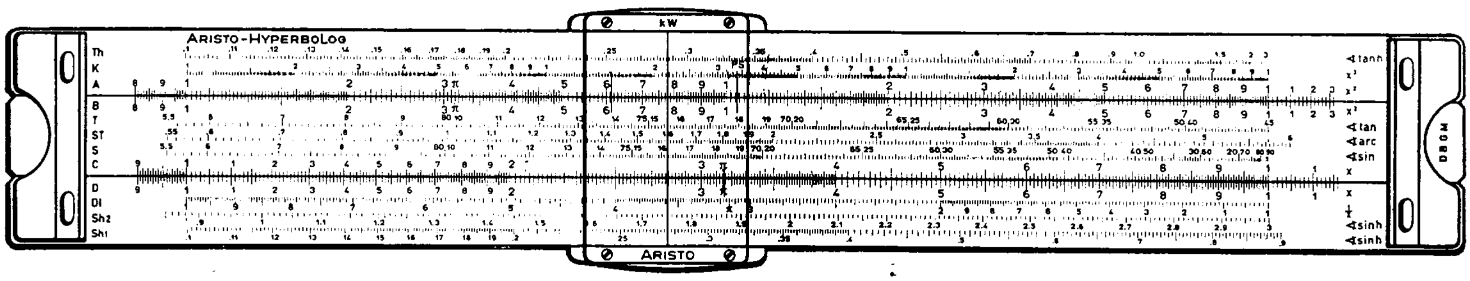

Fig. 5 Hyperbolic face

THE SLIDE RULE ARISTO HYPERLOG 0972

The LogLog face

Scale	Description	Function	Position
LL00	LogLog scale, range 0.999 to 0.989	$e^{-0.001x}$	On rule body
LL01	LogLog scale, range 0.99 to 0.9	$e^{-0.01x}$	
LL02	LogLog scale, range 0.91 to 0.35	$e^{-0.1x}$	
LL03	LogLog scale, range 0.4 to 10^{-5}	e^{-x}	
DF	Folded scale	πx	On the slide
CF	Folded scale	πx	
CIF	Scale of reciprocals of CF	$1/\pi x$	
L	Mantissa scale	lg x	
CI	Scale of reciprocals of C	1/x	
C	Fundamental scale	x	
D	Fundamental scale	x	
LL3	LogLog scale, range 2.5 to 10^5	e^x	On rule body
LL2	LogLog scale, range 1.1 to 3.0	$e^{0.1x}$	
LL1	LogLog scale, range 1.01 to 1.11	$e^{0.01x}$	
LL0	LogLog scale, range 1.001 to 1.011	$e^{0.001x}$	

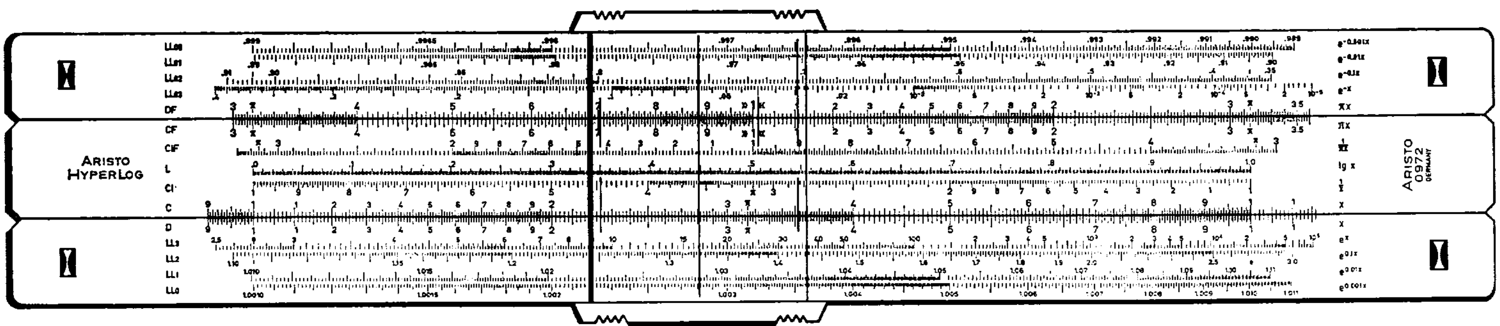

Fig. 6 LogLog face

The hyperbolic face

Scale	Description	Range	Function	Position
H2	Hyperbolic scale,	range 1.4 to 10	$\sqrt{1+x^2}$	On rule body
Sh2	Scale of hyperbolic sines,	range 0.85 to 3	∢ sinh	
Th	Scale of hyperbolic tangents,	range 0.1 to 3	∢ tanh	
K	Scale of cubes		x^3	
A	Scale of squares		x^2	
B	Scale of squares		x^2	On the slide
T	Scale of tangents and cotangents for angles	of 5.5° to 45°	∢ tan	
ST	Scale of tangents, sines and radian measure for angles	of 0.55° to 6°	∢ arc	
S	Scale of sines for angles	of 5,5° to 90°	∢ sin	
P	Pythagoras scale	range 0 to 0.995	$\sqrt{1-x^2}$	
C	Fundamental scale		x	
D	Fundamental scale		x	On rule body
DI	Scale of reciprocals of D		1/x	
Ch	Scale of hyperbolic cosines	range 0 to 3	∢ cosh	
Sh1	Scale of hyperbolic sines	range 0.1 to 0.9	∢ sinh	
H1	Hyperbolic scale	range 1.005 to 1.5	$\sqrt{1+x^2}$	

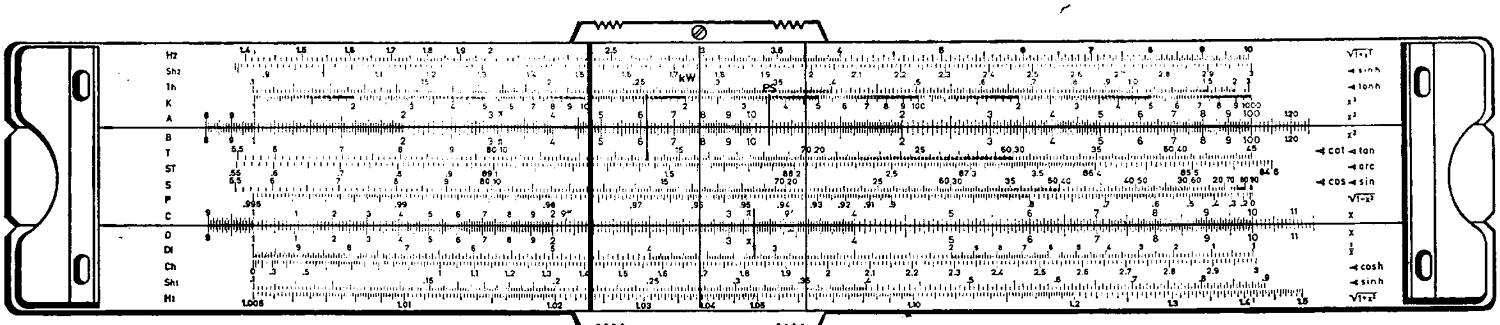

Fig. 7 Hyperbolic face

3. Reading the scales

To use the slide rule efficiently for rapid calculations is essentially a matter of learning to read the scales quickly and correctly. The figures 8—11 show examples referred to the most frequently used fundamental scales C and D. The principal intervals, marked by long strokes, are figured from 1 to 10 (fig. 8). The end mark 10 is, on the trigonometrical face, repeated as 1, because this graduation can be regarded as the beginning of another and identical scale.

Fig. 8 The main intervals

In the range between figured graduations 1 and 2 the scale resembles the graduation of a millimeter scale, the difference consisting only in the reduction of interval width, progressively from left to right and in the use, on the slide rule, of the initial mark 1 in place of 0.

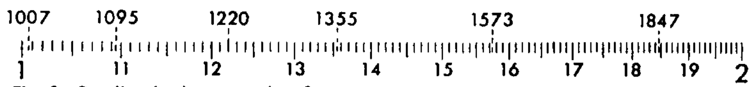

Fig. 9 Reading in the range 1 to 2

The graduation marked 2 of a millimeter scale can be considered as indicating 2 cm, 20 mm, 0.2 dm, 0.02 m and so on. In other words the dimension, marked 2, can be thought of in association with various powers of 10. Similarly, the figures on the slide rule scales are independent of the position of the decimal point. It is therefore advisable to read a series of figures without regard to the decimal point, expressing them as simple numbers, e. g., 1—0—4 and not as one hundred and four. This will avoid omitting figures. For practice, move the cursor slowly to the right, from the value marked 1 and read at each graduation line the series of numbers: 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, etc.

The cursor hairline is so thin, by comparison with the width of the intervals, that the midpoint of a subdivision (between two graduations) can easily be located. Indeed, smaller fractions of subdivisions can be distinguished by eye. With practice, even one tenth of a subdivision can be estimated and thus the fourth digit obtained.

For practice, move the cursor slowly still further to the right. Between the graduations 1310 and 1320 estimates can be made, e. g., as 1310, 1311, 1312, 1313, 1314, 1315, etc.

Between a numbered graduation and that immediately following it, especially at the beginning of the scale, observe that a zero is to be read, e. g., 1000, 1001, 1002, 1003, etc. (note 1007 in fig. 9).

Fig. 10 Reading in the range 2 to 4

Because the intervals to the left of the figure 2 are already very narrow, in the following range between figures 2 and 4, only every second interval is marked. This yields a new graduation pattern, in which from mark to mark the even values are to be counted off: 200, 202, 204, 206, 208, 210, 212, 214, etc. The midpoints of the intervals give the odd numbers: 201, 203, 205, 207, 209, 211, 213, etc. Fig. 10 shows some examples.

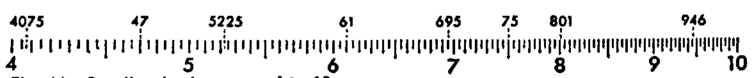

Fig. 11 Reading in the range 4 to 10

In the range 4 to 10, the intervals are marked in subdivisions of 5 units and the successive graduations are read as: 400, 405, 410, 415, 420, 425, 430 etc.

Intermediate values must be estimated. Midway between the marks 400 and 405 is the value 4025; a little to the left of this the value 402, a little to the right 403. In like fashion, at the midpoint of the next pair of subdivision marks is found the value 4075. Fig. 11 shows a series of such points.

4. Making approximations

It was explained, in chapter 3, that when using the slide rule, numbers are set or read as a simple series of digits. The correct position of the decimal point is determined by approximation. By this means, a check is at the same time imposed on the order of magnitude of the slide rule result.

Rules for approximation:

Values strongly rounded off!

Examples: $3.43 \approx 3$ $\quad 9.51 \approx 10$ $\quad 7.61 \approx 8$

When multiplying, round up one factor, round down the other!

Examples: $8.92 \times 127 \approx 10 \times 120 = 1200$

$2.19 \times 9830 \approx 2 \times 10000 = 20000$

When dividing, simplify!

Numerator and denominator are rounded off in the same direction.

Examples: $\frac{725}{539} = \frac{7.25}{5.39} \approx \frac{7}{5} = 1.4$

$$\frac{640 \times 15.3}{51 \times 0.8} \approx \frac{60 \times 20}{5 \times 1} = 240$$

Very large or very small numbers are simplified by separation of powers of 10.

Examples: $73215 \approx 7 \times 10^4$ $\quad 0.0078 \approx 8 \times 10^{-3}$

$89 \approx 9 \times 10^1$ $\quad 0.706 \approx 7 \times 10^{-1}$

Separation of powers of 10, when multiplying or dividing with very large numbers, gives a clearer appreciation of quantity.

Examples: $0.07325 \times 0.000513 \approx 8 \times 10^{-2} \times 5 \times 10^{-4} = 40 \times 10^{-6} = 4 \times 10^{-5}$

$$\frac{2950}{0.00598} \approx \frac{3 \times 10^3}{6 \times 10^{-3}} = 0.5 \times 10^6$$

5. The slide rule principle

Calculations are carried through by the mechanical addition or subtraction of scale lengths. The process can be very simply explained by considering two abutting millimeter scales, sliding one upon the other. Fig. 12 shows the example $2 + 3 = 5$. If the initial mark 0 of the upper scale is moved over the value 2 of the

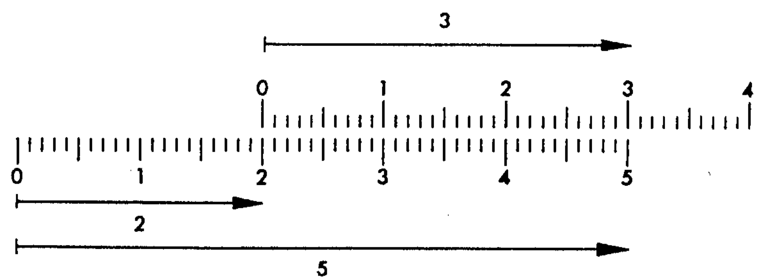

Fig. 12 Graphic addition by use of two ordinary scales

lower scale, then immediately under 3 of the upper scale is found the sum 5 on the lower scale. In addition the sum 2 + 1 = 3 or 20 + 15 = 35 can be read from fig. 12, if the millimeters are counted off.

The subtraction 5 — 3 = 2 can also be read from fig. 12, by reversing the process described above. From the length 0 — 5 on the lower scale the length 0 — 3 on the upper scale is subtracted by setting the values 3 and 5 of the upper and lower scales, respectively, the one over the other and reading the result 2 from the lower scale under the initial mark 0 of the upper scale.

In the slide rule the graduations are disposed upon a rigid rule body and on a slide moving therein. The scales of the slide rule are, however, logarithmically divided and so the addition of two scale lengths performs a multiplication and a subtraction of two lengths carries out a division.

6. Multiplication

(Two scale lengths are added.)

The initial value 1, the left hand index of scale C of the slide, is brought over the value 18 on scale D. By moving the cursor to the value 13 on scale C we add the length 13 to the length 18. The product 234 can be read under the cursor hairline on scale D. The position of the decimal point can be located by an approximation, (20 × 10 = 200).

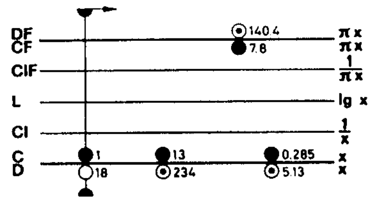

Fig. 13 18 × 13 = 234
18 × 0.285 = 5.13
18 × 7.8 = 140.4

To read the product of 18 × 7.8, the slide would have to be traversed, that is, the terminal index 10 of scale C would be brought over the factor 18 on D. With the ARISTO HyperboLog and ARISTO HyperLog, this additional slide setting can be avoided, if the upper pair of scales CF/DF is used for the multiplication.

Scales CF and DF make this simplification possible, because they are a repetition of the fundamental scales C and D, with the difference that the initial index 1, is placed approximately in the middle of the rule. If, for example, the value 1 of scale C is placed opposite 18 on scale D, the upper scale pair will show an identical setting, i. e., 1 on scale CF beneath 18 on DF. Multiplication by the factor 18 can be carried out on either scale pair. The product 18 × 7.8 can be found on scales CF/DF by moving the cursor over 7.8 on scale CF and reading 140.4 on scale DF.

7. Division

(Subtraction of one scale length from another. This is the reverse of multiplication.)

The cursor hairline is brought over the value 2620 on scale D and the value 17.7 on scale C moved under the cursor line. The two values are then in juxtaposition. The quotient 148 is read on D under the left hand index of C of the slide. In other cases, the quotient may be read under the right hand index of the slide.

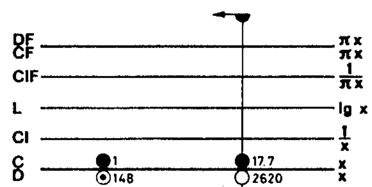

Fig. 14 2620 ÷ 17.7 = 148
Roughly: 3000 ÷ 20 = 150

Naturally, over 1 on CF the quotient can be read on DF, because the division 2620 ÷ 17.7 has also been set on this scale pair. In division with scales CF/DF, the factors are in the same relative position as written in a vulgar fraction.

The slide setting is identical with that for the multiplication 148 × 17.7 = 2620. The difference between multiplication and division consists only in the order of setting and reading. After setting up the division, the quotient will in any case be read on the body scale, under the left hand or right hand index; slide re-setting will not be necessary. This characteristic feature will be used in the following chapters.

8. The folded scales CF and DF

In graduation pattern, scales CF and DF are identical with the fundamental scale C and D, but are laterally displaced, with respect to the fundamental scales, by the scale length corresponding to the value of $\pi = 3.142$. The value figured 1, of these folded scales lies near the middle of the rule, producing an overlapping of the fundamental scales by half the rule length. The two pairs of scales, C/D and CF/DF, constitute a working assembly achieving advantages in multiplication, division, tabulation and proportion problems.

Index 1 of scale CF stands opposite, on DF, the same value as is matched with index 1 or 10 of scale C on D. Any of the multiplications discussed earlier can be begun on the scale pair CF/DF, with advantage, since the initial setting can always be chosen at once. It is not then necessary to decide whether the initial or final scale index should be used. If a division is set with the upper scale pair, the numerator and denominator are in their customary relative position, with the parting line between the scales corresponding to the division line in the fraction as written.

If the result of a problem cannot be read from one scale pair, it is always possible to find it from the other pair and slide re-setting is avoided. The yellow strips on the slide are a reminder that factors taken on the moveable slide scales C or CF yield results to be read on D under C or on DF over CF.

8.1 Tabulation without slide re-setting

$y = 29\,x$

x	1.7	3.45	5.0	10
y	49.3	100	145	290

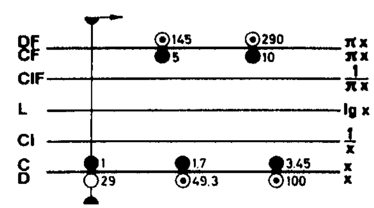

Fig. 15

For x = 5 the upper pair of scales CF and DF provides the answer without slide re-setting.

$$y = \frac{28.2}{x} = 28.2 \times \frac{1}{x}$$

x	7.43	2.92	1.567
y	3.795	9.66	18.0

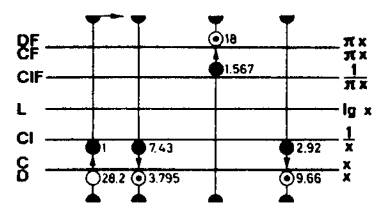

Fig. 16

$$y = \frac{x}{18.2} = \frac{1}{18.2} \times x$$

x	3.17	112.1
y	0.1742	6.16

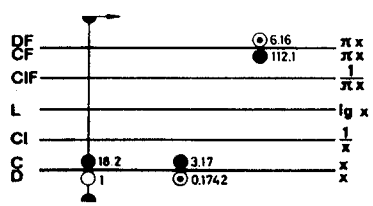

Fig. 17

8.2 Direct reading of products and quotients involving π

A further advantage issues from the displacement of scales CF and DF by the value $\pi = 3.142$. By switching from D to DF or from C to CF, multiplication by π is performed automatically. Conversely, a division by π is accomplished by changing from DF to D or CF to C. If, for example, a diameter d is set with the cursor on D, the circumference π d can be read at once on DF. Similarly, the angular velocity $\omega = 2\pi f$ is found on DF when 2 f is set on D.

The possibility of taking the final reading by switching scales should always be considered when dealing with problems involving the factor π. Fig. 18 shows a range of results incorporating π, demonstrating the possibilities of a single cursor setting. Compare with section 24.1, for work with the factor 360.

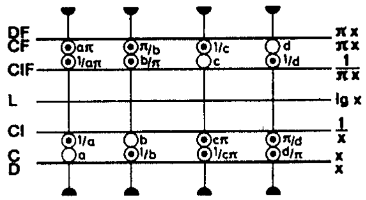

Fig. 18 Calculations with π

9. Combined multiplication and division

In solving expressions of the form $\frac{a \times b}{c}$ the rule to apply is:

First divide, then multiply.

The intermediate result of the division of 345 by 132 in fig. 19 need not be read. The slide rule scales are positioned ready for the final multiplication. The cursor is moved over the value 22 on scale C and the result read on scale D, viz. 57.5.

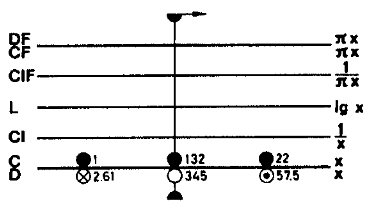

Fig. 19

$$\frac{345 \times 22}{132} = 57.5$$

Roughly: $\frac{300 \times 20}{100} = 60$

If, in this example, the further factor 19.5 is introduced, we have

$$\frac{345 \times 22}{132 \times 19.5} = 2.95$$

The solution obtained in fig. 19 is divided by moving the value 19.5 on scale C under the cursor hairline, thus dividing 57.5 by 19.5. Should there be, in examples of this type, yet more factors in numerator and denominator, simply divide and multiply alternately. The rhythmical alternation between slide and cursor positioning leads to smooth flowing calculation with minimal setting.

In such problems, it can happen that the slide, following a division, projects too far out of the rule body to permit a setting. To perform multiplication, the slide must be traversed. By careful choice of setting for division, between scales C/D or CF/DF, the necessity for slide traversing can often be avoided.

10. Reciprocal scales CI and CIF

Scale CI is divided exactly as are the fundamental scales C and D, but the intervals progress in the opposite direction, i. e., from right to left. To obviate errors, the figuring of the graduations is in red.

If the cursor is set to any value x on scale C, the reciprocal 1/x can be read from CI, as indicated by the scale identification symbol at the right hand end of the scale. Over 5 on C is 1/5 = 0.2 on CI. Of more importance, however, is the fact that the reciprocal scale can be used in the reverse direction. By switching from CI to C we find, e. g., under 4 on CI the value 1/4 = 0.25 on C.

The occasional use of scale CI to find reciprocals would not justify its provision; its real value lies in the fact that it can be used to avoid many settings in complex examples.

$$\frac{4}{5} \text{ can be written as } 4 \times \frac{1}{5} \text{ and } 4 \times 5 \text{ as its equivalent } \frac{4}{1/5}$$

Whilst these expressions are perhaps unusual, they offer the advantage, for slide rule working, of converting a division into a multiplication or, conversely, a multiplication to a division. The value of this will best be shown by a "game" with simple numbers:

1. With the cursor set to 6 on D, bring 2 on C under the hairline. We then have the usual setting for the division of 6 by 2 (fig. 20). If however, the cursor is left in place and by a movement of the slide, 2 on CI is brought under the hairline, we have the multiplication 6 × 2 and read the product, 12, as for a division, under the index of the slide (fig. 21). Actually, we have found the quotient of 6 ÷ 0.5, because simultaneously with bringing 2 on CI under the hairline, the reciprocal 0.5 on C was set to the cursor.

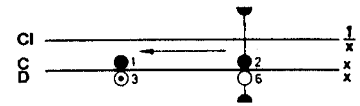

Fig. 20

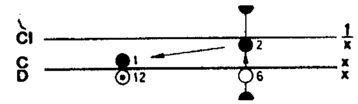

Fig. 21

2. Now, letting the index 1 of C remain over 12 on D, move the cursor to 4 on C, establishing the normal setting for the multiplication 12 × 4 = 48 (fig. 22). By moving the cursor to 4 on CI, however, we can read the quotient of 12 ÷ 4 on D. i. e., 3. (fig. 23). In other words, because under 4 on CI stands its reciprocal 1/4 = 0.25 on C, we actually calculate 12 × 0.25 = 3.

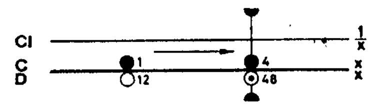

Fig. 22

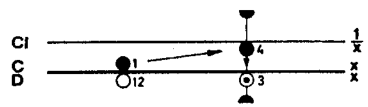

Fig. 23

Thus there are two setting possibilities in multiplication and division and the experienced operator will chose the best, in the solution of a complex example by alternate division and multiplication.

The stated relationship between scales C and CI holds similarly between scales CF and CIF. To show that this is so, the "number game" can usefully be replayed with the scale group CF/DF/CIF. Anyone who thoroughly studies the foregoing will at once recognise that scale CIF is the logical complement of the scale system. Whoever properly exploits the advantages of the folded scales will use scale CIF as often as scale CI.

Expressions of the form a × b × c or $\frac{a}{b \times c \times d}$ etc. will be solved by alternate multiplication and division, as shown in section 9 on combined multiplication and division. In the course of the calculation with scales C, D and CI switching to the scale group CF, DF and CIF will avoid slide traversing in multiplication.

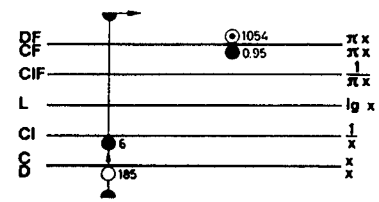

Fig. 24

185 × 6 × 0.95 = 1054

Roughly: 200 × 6 × 1 = 1200

In the example of fig. 24, the factors 185 on scale D and 6 on scale CI are set in opposition, as for a division. Multiplication by 0.95 is then carried out with the upper scale CF and the result 1054 read on DF over 0.95 on CF.

10.1 Reciprocal scale DI

This scale of reciprocals, DI, enables the experienced slide rule user, on occasion, to switch from body- to slide scales; for example, when working with proportions. Practice with simple multiplication and division, using scales C and DI, is recommended.

11. Proportions

Proportions of the form $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ are particularly simple to calculate with the slide rule because, after setting one ratio, all other equal ratios can be found by moving the cursor. The parting line between body- and slide scales can be regarded as the line in a common fraction, as written. Proportions should preferably be expressed in this form.

Example: 9.5 lb of a given material cost \$ 6.3. What will be the cost of 8.4 lb?

The solution by "rule of three" follows from $\frac{6.30}{9.5} \times 8.4 = 5.57$

The calculation can be more conveniently made if the ratio of weight and price is set up as a proportion. If the given weight on DF is brought over the corresponding price \$ 6.30 on CF, all equivalent weight/price ratios will be shown on scales DF/CF and D/C. On scales DF and D are all weights, in accordance with the initial setting and on scales CF and C are the corresponding prices. Opposite the weight 8.4 lb is found the price \$ 5.57. Other weight/price relationships are shown in fig. 25.

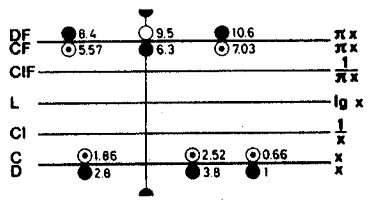

Fig. 25 Proportion

10.6 lb cost \$ 7.03 (scales CF/DF)
3.8 lb cost \$ 2.52 (scales C/D)
2.8 lb cost \$ 1.86 (scales C/D)
1 lb cost \$ 0.66 (scales C/D)

The proportion can be extended at will:

$$\frac{\text{lb}}{\$} = \frac{9.5}{6.3} = \frac{8.4}{5.57} = \frac{10.6}{7.03} = \frac{3.8}{2.52} = \frac{2.8}{1.86} = \frac{1}{0.66} = \ldots$$

Calculation by proportions proceeds independently of the earlier mentioned rule. It is of no consequence, when and how the weight/price ratio is set up, the only difference arising is that weights are looked for on the scale on which the first weight was set and the corresponding prices on the adjacent scale. In the example above 6.3 could have been set on scale DF and 9.5 on CF. The price 8.4 would then be found on CF and the required proportion read on DF, as 5.57.

This principle of direct proportionality, a : b = c : d, applies with equal force to indirect proportion, which leads to the identity $a \times b = c \times d$, to be solved with the aid of the reciprocal scales. Finally, the principle can be seen applicable to the "mixed" proportions $a \times b = c : d$ and $a : b = c \times d$.

12. The scales A, B and K

If the cursor line is brought over any value x on scale C, the value x^2 can be found on scale B (the scale of squares) or x^3 on scale K. Conversely, the square root or the cube root can be obtained.

a) $2^2 = 4$ $\quad 2^3 = 8$

b) $32.7^2 = 3.27^2 \times 10^2 = 1070$
$\quad 32.7^3 = 3.27^3 \times 10^3 = 35000$

c) $\sqrt[2]{9} = 3$ $\quad \sqrt[3]{27} = 3$

d) $\sqrt[2]{51} = 7.14$ $\quad \sqrt[3]{364} = 7.14$

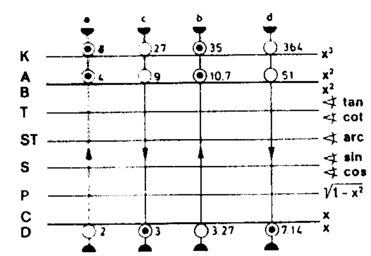

Fig. 26 Powers and roots

The position of the decimal point is best found by approximation. In calculating powers and roots it is of advantage to work in powers of ten, to obtain numbers in which the position of the decimal point is easily seen. To this end, the scale of squares is figured 1 to 100 and the cube scale 1 to 1000. The range in which the cursor is to be set follows from the figuring of the scale.

Examples:

$\sqrt{3200} = \sqrt{32 \times 100} = 10 \times \sqrt{32} = 10 \times 5.66 = 56.6$ Separating factor 10^2

$\sqrt[3]{0.1813} = \sqrt[3]{\frac{181.3}{1000}} = \frac{1}{10} \times \sqrt[3]{181.3} = \frac{1}{10} \times 5.66 = 0.566$ Separating factor 10^3

12.1 Calculation with the scales A and B

Scales A and B, like the fundamental scales C and D, are identically divided, with the difference that they consist of two scale segments, each half the length of the fundamental scales C and D. The left hand segment is figured 1 to 10 and the right, 10 to 100. All examples so far discussed can be solved with the scales A and B, by methods described for the fundamental scales. The reading will be somewhat less, because the graduations are disposed over only half the length of the rule.

The adjacent scale arrangement offers the great advantage that slide re-setting is no longer a necessity.

In many cases it is convenient, if a problem begins with a squared factor, to continue the calculation on the scale of squares.

13. The Pythagoras scales

(ARISTO HyperLog only)

13.1 Scale P

In a right triangle, with hypotenuse 1, the Pythagoras relationship with the other two sides holds.

$$y = \sqrt{1 - x^2}$$

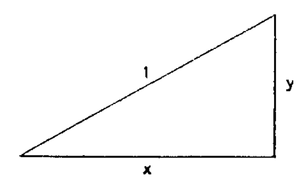

Fig. 27

For any setting of x on the fundamental scale D we find the value of y on scale P and, conversely, $x = \sqrt{1 - y^2}$ on D if y is set on P. In the example of fig. 28 it is clear that 0.6 could equally well be set on D or on P. In either case the required value 0.8 is found on the corresponding adjacent scale.

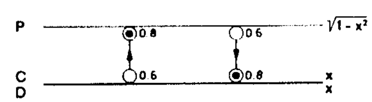

Fig. 28 $\sqrt{1 - 0{,}6^2} = 0.8$

The choice of initial setting should be made with due regard to maximum accuracy of reading. In the example $\sqrt{1 - 0.15^2} = 0.9887$ the factor 0.15 will be set on C.

The relationship holds only for the given range in scale P. Should scale DI project too far, the alternative relationship shown in fig. 29 will apply.

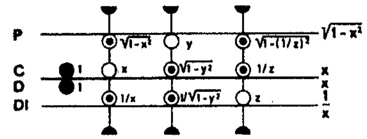

Fig. 29

Scale P simplifies, in conjunction with scales C and S, the conversion

sin ←→ cos, because in right triangles

$$\sin^2 \alpha + \cos^2 \alpha = 1.$$

The trigonometrical solution is often more elegant — (see section 16).

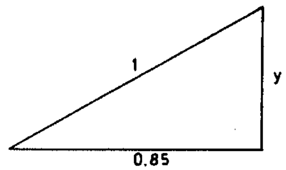

Fig. 30

Example in electrical engineering:

apparent load $\triangleq$ 1.0

effective load $\triangleq$ 0.85

wattless load $\triangleq \sqrt{1 - 0.85^2} = 0.527$

To achieve greater accuracy in calculations with the scales of squares, rearrangement of the data is useful. For example:

$$\sqrt{0.91} = \sqrt{1 - 0.09} = 0.9540$$

The factor 0.09 is taken on the left hand portion of scale A. On D is then found $\sqrt{0.09} = 0.3$ and the value of $\sqrt{1 - 0.3^2} = 0.9540$ is seen on P. Greater accuracy is obtained, in this way, for roots greater than about $\sqrt{0.65}$ and is always convenient when the radicand is close to 0.01, 1, 100, etc.

13.2 Scales H1 and H2

The two part scale H, identified as scale $\sqrt{1 + x^2}$ at the right hand extremity has, like scale P, a direct relationship with fundamental scale D. With x taken on scale D, the value of $y = \sqrt{1 + x^2}$ is found at once on scale H. Conversely, when y is set on scale H, we read the value $x = \sqrt{y^2 - 1}$ on scale D. Scale H1 covers the range 0.1 to 1.0 on D whilst H2 has values corresponding with 1 to 10.

The relationship between scales H1 and H2 with scale P enables Pythagoras calculations to be solved:

With scales H1 and H2: $c = a \cdot \sqrt{1 + \left(\frac{b}{a}\right)^2}$

With scale P: $b = c \cdot \sqrt{1 - \left(\frac{a}{c}\right)^2}$

In section 19.2 will be shown a further application of these scales in association with scales Sh1 and Sh2, based on the relationships:

$$\cosh x = \sqrt{1 + \sinh^2 x} \quad \text{and} \quad \sinh x = \sqrt{\cosh^2 x - 1}$$

Reference is made also to the identity $\sec \alpha = \sqrt{1 + \tan^2 \alpha}$. Except for a change of scale identifications, scales H and D show in opposition coordinate values $y = \sqrt{x^2 - 1}$ and $x = \sqrt{1 + y^2}$ of the unit hyperbola $x^2 - y^2 = 1$.

14. Trigonometrical functions

All angle functions are referred to the fundamental scale D and the angular scales, in the 360° system, are decimally divided.

If an angle is set with the cursor on scales S or T, the corresponding function value can be found under the cursor line on scale D. Conversely, for a function value set on scale D, the corresponding angle can be read on scale S or T.

The figuring of the decimally divided scales S and T applies uniquely to the inscribed angle values.

The slide rule gives the function value for angles in the first quadrant only. The relationships for any angle, with those of an angle in the first quadrant, are tabulated below.

	$\pm \alpha$	$90^\circ \pm \alpha$	$180^\circ \pm \alpha$	$270^\circ \pm \alpha$
sin	$\pm \sin \alpha$	$+ \cos \alpha$	$\mp \sin \alpha$	$- \cos \alpha$
cos	$+ \cos \alpha$	$\mp \sin \alpha$	$- \cos \alpha$	$\pm \sin \alpha$
tan	$\pm \tan \alpha$	$\mp \cot \alpha$	$\pm \tan \alpha$	$\mp \cot \alpha$
cot	$\pm \cot \alpha$	$\mp \tan \alpha$	$\pm \cot \alpha$	$\mp \tan \alpha$

14.1 Sine scale S

The scale of sines S is figured between 5.5° and 90° in black and also in red from right to left, for cosines between 0° and 84.5°. All sines and cosines read on D are prefixed with 0 before the decimal point.

Examples:
a) sin 30° = 0.500
b) sin 26° = 0.438
c) cos 75° = 0.259
d) cos 42.8° = 0.733

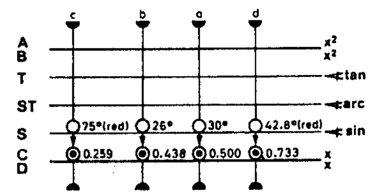

Fig. 31 Sine and cosine with 0971

14.1.1 Sines of angles $\alpha > 45^\circ$ (ARISTO HyperLog only)

are read with enhanced accuracy on the red figured scale P, using the identity $\sin \alpha = \sqrt{1 - \cos^2 \alpha}$. To set the angle, the red figures of scale S are used, hence the colour rule for sine functions: set and read sine functions in like colours.

For cosines of angles $\alpha < 45^\circ$, an analogous colour rule follows from $\cos \alpha = \sqrt{1 - \sin^2 \alpha}$. For every setting on scale S read in the contrasting colour the function value on scale D or P.

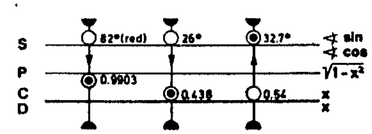

Fig. 32a Sine with 0972

Examples:

sin 26° = 0.438

sin 82° $= \sqrt{1 - \cos^2 82^\circ} = 0.9903$

arc sin 0.54 = 32.7°

cos 75° = 0.2588

cos 7° $= \sqrt{1 - \sin^2 7^\circ} = 0.99255$

arc cos 0.9852 = 9.87°

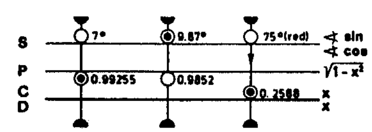

Fig. 32b Cosine with 0972

14.2 Tangent scale T

The scale of tangents is figured, in black, from 5.5° to 45° and counter-clockwise in red from 45° to 84.5°. Angles read in black have function values, prefixed by 0 . . . , on scale C.

As $\tan \alpha = 1/\tan(90° - \alpha)$, function values for angles $\alpha > 45°$ in red figures can either be read on scale CI or, by putting the slide in the "neutral" position, on scale DI. The scale of reciprocals therefore covers the range $\tan 45° = 1$ to $\tan 84.5° = 10$.

Examples:

tan 14°	=	0.249
tan 23.6°	=	0.437
tan 41.1°	=	0.872
tan 51.2°	=	1.244
tan 73.4°	=	3.35
tan 81.4°	=	6.61
arc tan 1.75	=	60.25°
arc tan 2.0	=	63.43°

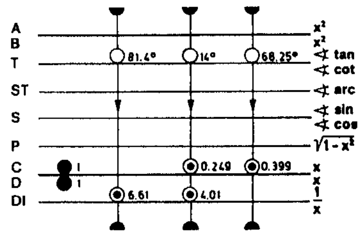

Fig. 33 Tangent and cotangent

cot 9°	=	6.31
cot 14°	=	4.01
cot 23.6°	=	2.289
cot 41.1°	=	1.146
cot 51.2°	=	0.804
cot 68.25°	=	0.399
cot 77°	=	0.2309
arc cot 2.0	=	26.57°
arc cot 1.75	=	29.74°

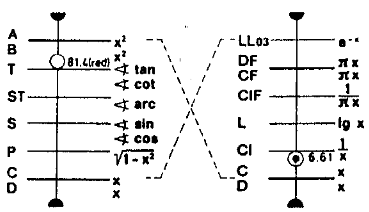

Fig. 34 Reading tan 81.4 on the scale CI

Values of cotangents are read as reciprocals of tangent values, using the identity $\cot \alpha = 1/\tan \alpha$. Thus we find cotangent values of angles < 45° on scale CI or DI and those for angles > 45° on scale C. Note: Like colours for setting and reading give tangent values, unlike colours the values of the cotangents.

15. Scale ST

This scale is an extension of scale S and T for angles, the function value of which is between 0.01 and 0.1, read on scale C. It provides, at the same time by going from scale ST to C, for the important task of conversion between circular and radian measure.

If $\sin \alpha$ and $\tan \alpha$ for $\alpha < 5.5°$ or $\cos \alpha$ and $\cot \alpha$ for $\alpha > 84.5°$ are to be found, use the relationship:

$$\sin \alpha \approx \tan \alpha \approx \cos(90° - \alpha) \approx \cot(90° - \alpha) \approx \frac{1}{180}\alpha° \approx 0.01745\,\alpha$$

Scale ST is figured between 0.55° and 6° but is subdivided in radian measure. This makes possible accurate reading on scale D in radians for sine and tangent functions of small angles. The red figuring of scale ST, from right to left, between 84° and 89.45° enables the scale of small angles to be used for the cosines and cotangents of large angles.

The agreement in value between $\sin \alpha$, $\tan \alpha$, and $\text{arc}\,\alpha$ is very good up to 4° for example sin 4° = 0.0698, tan 4° = 0.0699, and arc 4° = 0.0698. For larger angles between 4° and 6° more accurate values can be obtained from the relationship:

$$\sin \alpha = \alpha \times \frac{\sin 6°}{6°} \quad \text{or} \quad \tan \alpha = \alpha \times \frac{\tan 6°}{6°}$$

15.1 Small angles — large angles

A few examples of these approximations will be useful exercises in the application of the function scales:

Examples:

$\sin 4.7° = 4.7 \times \frac{\sin 6°}{6°} = 0.0819$ $\qquad$ $\tan 4.7° = 4.7 \times \frac{\tan 6°}{6°} = 0.0822$

$\sin 5.3° = 5.3 \times \frac{\sin 6°}{6°} = 0.0924$ $\qquad$ $\tan 5.3° = 5.3 \times \frac{\tan 6°}{6°} = 0.0928$

Values $\cos \alpha$ for $\alpha < 5.7°$ and $\sin \alpha$ for $\alpha > 84.3°$ can only be read rather inaccurately from the rule. In such cases, the first term of an expansion provides a helpful approximation:

$$\cos \alpha = 1 - \frac{\alpha^2}{2} \ (\alpha \text{ in rad})$$

Example:

$$\cos 1.5° = 1 - \frac{0.0262^2}{2} = 1 - \frac{0.000686}{2} = 1 - 0.000343 = 0.999657$$

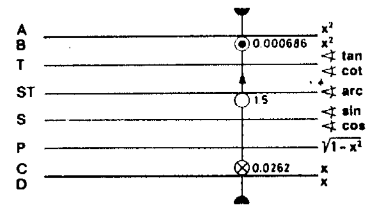

Fig. 35 cos 1.5° = 0.999657

To find the second term of the expansion the angle 1.5° on scale ST is set with the cursor. On scale C is then the radian measure of the angle and on scale B the square of this value, i. e., 0.000686. Division by 2 is done mentally. Finally, the subtraction is performed.

Example: $\sin 86.5° = \cos 3.5° = 1 - \frac{0.0611^2}{2} = 0.99813$

15.2 Conversion circular ←→ radian measure

Conversions between circular and radian measure involve the relationship

$$\frac{\alpha}{b} = \frac{180°}{\pi} = \frac{360°}{2\pi}$$

and are obtained by a cursor setting only, because scale ST is a fundamental scale laterally displaced by $180/\pi$. For any angle on scale ST, the radian measure is read on scale C, beginning with 0.0 . . . Opposite 1° on scale ST stands 0.01745, i. e., $\pi/180$, on scale C. Conversely, for any value in radians the angle can be found. This holds, not merely for all angles marked on scale ST but also, because the ST scale is decimally subdivided, for all angles — 1 can be read as 0.1°, 10° and so on. In consequence, it is only necessary to change the position of the decimal point in the figures for radian measure (see fig. 36).

Examples:

a) 0.1° = 0.001 745 rad
b) 10° = 0.1745 rad

c) 5° = 0.08725 rad
d) 50° = 0.8725 rad

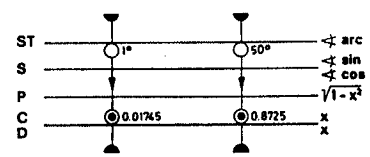

Fig. 36

If the small angle is given in seconds or minutes, first convert to a decimal fraction of a degree: 1′ = 1/60° and 1″ = 1/3600° (see also sections 15.3 and 24.1). By setting 6 or 36 on scale DF to 1° on scale ST, a proportion is conveniently established for all such conversions.

15.3 The marks ϱ' and ϱ''

These gauge marks simplify conversion when the angle is given in minutes or seconds of arc. They indicate the factors:

$$\varrho' = \frac{180}{\pi} \times 60 = 3438 \qquad \text{for minutes}$$

$$\varrho'' = \frac{180}{\pi} \times 60 \times 60 = 206265 \quad \text{for seconds}$$

Hence, converting by division: $\text{arc}\,\alpha = \frac{\alpha'}{\varrho'} = \frac{\alpha''}{\varrho''}$

For example:

$$\text{arc } 22' = \frac{22'}{\varrho'} = 0.00640 \text{ rad}$$

$$\text{arc } 400' = \frac{400'}{\varrho'} = 0.1163 \text{ rad}$$

$$\text{arc } 17'' = \frac{17''}{\varrho''} = 0.0000824 \text{ rad}$$

$$\text{arc } 380'' = \frac{380''}{\varrho''} = 0.001843 \text{ rad}$$

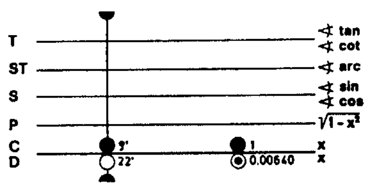

Fig. 37 arc 22′ = 0.00640 rad

These marks are of great use when finding small angles or lengths of arc for given radii:

$\alpha = \frac{b}{r} \times \varrho$ when the angle is to be found.

$b = \frac{\alpha \times r}{\varrho}$ when the length of arc is required.

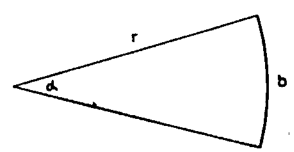

Fig. 38

Examples:

$$\alpha = \frac{0.6}{45} \times \varrho' = 45.8'$$

$$b = \frac{48'' \times 67}{\varrho''} = 0.0156$$

16. Trigonometrical solution of plane triangles

The advantage offered by the trigonometrical scales is not simply the availability of function values. Of more importance, function values can be used in calculation without their being read from the scales.

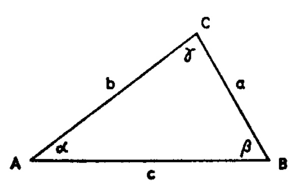

Fig. 39

The law of sines is a convincing example of the efficiency of the slide rule in solving proportions:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

When one of these ratios is set up by bringing the length on scale C opposite the corresponding angle on scale S or ST, all other parts of the triangle can at once be read.

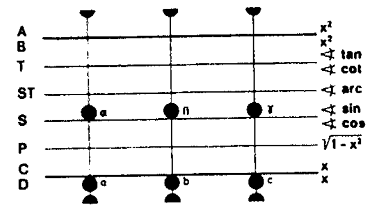

Fig. 40

In practice this law is most often applied in the case of right triangles, in which we have $\gamma = 90°$, $\sin\gamma = 1$, angle $\alpha = (90° - \beta)$ and angle $\beta = (90° - \alpha)$. The law of sines is then rearranged as

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = \frac{a}{\cos\beta} = \frac{b}{\cos\alpha}$$

and further: $\tan\alpha = \frac{a}{b}$

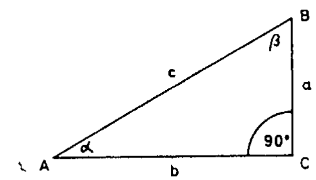

Fig. 41

Depending on the given elements, there follows one of two procedures:

1. Given any two parts (other than those of case 2).
2. Given the two short sides a and b.

Example of case 1:

Given: c = 5, b = 4.

Required a, α, β

$$\frac{5}{1} = \frac{4}{\sin 53.15°} = \frac{3}{\cos 53.15°}$$

$\beta = 53.15°$

$\alpha = 90° - 53.15° = 36.85°$

a = 3

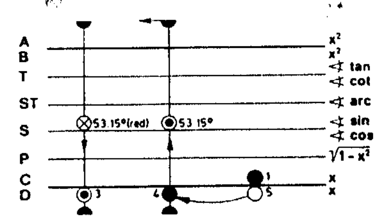

Fig. 42 Given the hypotenuse

Over short side 4 on scale D stands $\beta = 53.15°$ on scale S (black figures). It is immaterial whether we set next 90° − 53.15° = 36.85°, using black figures (sin) or set 53.15° on the red figures (cos). On scale D we find the other short side, 3.

If a short side and an angle are given, the calculation is begun by setting the side value to its opposite angle. Subsequent working is analogous to fig. 42 and the hypotenuse is found under the slide index on scale D.

It is at times convenient to set the angle on scale DF instead of scale D, to avoid slide re-setting. If this is done, all other sides are found at once on scale DF and the method needs no modification.

Example of case 2:

Given: a = 3, b = 4.

Required c, α, β

Angle α can be found from

$$\tan\alpha = \frac{a}{b} = \frac{3}{4}$$

or better, in the proportional form:

$$\frac{4}{1} = \frac{3}{\tan\alpha}.$$

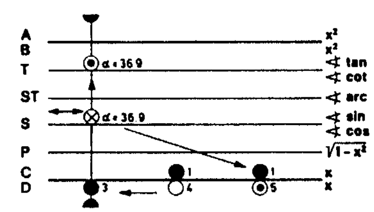

Fig. 43 Given the two sides

After setting the slide index to 4 on scale D, the cursor is brought over 3 on D. The angle 36.9° can then be read on scale T. In the second stage of the calculation, the sine rule is used: $\frac{1}{c} = \frac{3}{\sin 36.9^\circ}$

The cursor is already over 3 on scale D. The slide is moved to bring 36.9° on scale S under the hairline, when $c = 5$ can be read on D under the slide index.

For $\alpha > 45^\circ$, if $a > b$, the calculation proceeds just as easily. The work always begins with the larger of the short sides, but the complementary angle (in red figures) is read from scale T and correspondingly, the cosine must be set with the red figures of scale S.

Calculation will be simplified if a diagram is drawn, and error avoided. By cultivating this habit, one can deal only with angles $< 45^\circ$ or, if need be, with the complementary angle.

These two cited procedures for the solution of right triangles have special significance in connection with coordinate and vector calculations and in work with complex numbers. They apply in problems of conversion from rectangular coordinates to the polar form and vice versa.

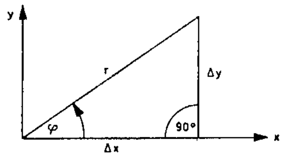

Fig. 44 $\Delta x, \Delta y \longleftrightarrow r, \underline{/\varphi}$

16.1 Complex numbers

Complex numbers in the coordinate form $Z = a + i\,b$ can easily be added or subtracted. The vector form $Z = r \times e^{i\varphi} = r\underline{/\varphi}$ is better suited to multiplication and division. For this reason the conversion of one form into the other must often be performed.

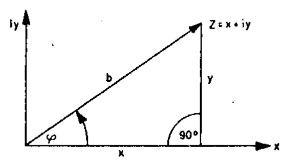

Fig. 45 $Z = x + i\,y = r\underline{/\varphi}$

Examples:

$Z = 4.5 + i\,1.3 = 4.68\underline{/16.13^\circ}$

$Z = 6.7\underline{/49^\circ} = 4.39 + i\,5.05$

The process of solution is shown in fig. 45 and follows the explanation given above.

17. The LogLog Scales

The LogLog scales are divided as logarithms of logarithms and are referred to the fundamental scales C and D. In the ARISTO HyperLog the range 10^{-5} to 10^{+5} is displayed in eight sections, four with negative exponents, e^{-x}, identified as LL00, LL01, LL02, LL03 from 10^{-5} to 0.999 and four with positive exponents, e^x, marked LL0, LL1, LL2, LL3 for 1.001 to 10^5. In the ARISTO HyperboLog the scales LL00 and LL0 are not included; the range 10^{-5} to 10^5 is displayed in six sections only, three with negative exponents and three with positive exponents.

Readings taken on the LogLog scales are unique values, that is to say, the value, e. g., 1.35 denotes only 1.35 and cannot be read as 13.5 or 135, as on the fundamental scales.

The LogLog scales LL and LL0 are reciprocal one of the other. They permit direct readings of reciprocals of numbers less than 2.5 with greater precision than is possible with scales CI or CIF.

Example: $\frac{1}{1.0170} = 0.98328$

By means of the exponential scales problems of involution or evolution are solved by addition or subtraction, respectively, of scale lengths. Thus, required powers, roots, and logarithms within the scale range can be calculated.

17.1 Powers and roots with exponents 10 and 100

The relative position of the LogLog scales is such that, by passing from one scale to that adjacent to it, the tenth power or the tenth root of a number set on one scale can at once be read on the other, depending on the direction in which the reading is made.

The relationships developed are shown by the examples of fig. 46, for cursor settings to the value 1.015 on scale LL1.

Examples:	Read on Scale
$1.015^{0.1} = \sqrt[10]{1.015} = 1.00149$	LL0
$1.015^{1} = 1.015$	LL1
$1.015^{10} = 1.1605$	LL2
$1.015^{100} = 4.43$	LL3
$\frac{1}{1.015^{100}} = 1.015^{-100} = 0.2257$	LL03
$\frac{1}{1.015^{10}} = 1.015^{-10} = 0.8617$	LL02
$\frac{1}{1.015^{1}} = 1.015^{-1} = 0.98522$	LL01
$\frac{1}{1.015^{0.1}} = \frac{1}{\sqrt[10]{1.015}} = 0.99851$	LL00

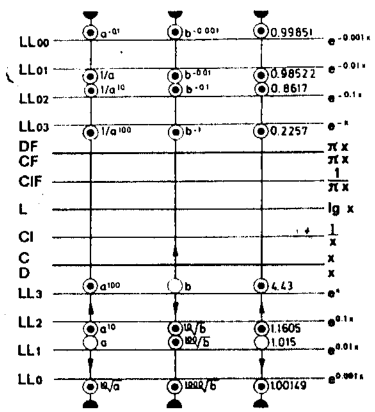

Fig. 46 LL scale assembly

Varied readings of the series shown in fig. 46:

$\sqrt[10]{4.43} = 1.1605 \quad \sqrt[10]{0.2257} = 0.98522 \quad 0.98522^{10} = 0.8617 \quad 1.00149^{1000} = 4.43$

These examples, although seldom met in practice, will serve to convey a better understanding of the construction of the LogLog scales.

17.2 Powers $y = a^x$

Involution or the raising of a number to a given power is carried out with the LogLog scales in association with the fundamental scale C in a manner analogous to multiplication with the fundamental scales.

Procedure:

a) Use the cursor to set the initial or terminal index of scale C to the base "a" on the appropriate LL scale. E. g., $a = 3.2$ in scale LL3.

b) Bring the cursor hairline over the value of the exponent x, read on scale C.

c) Read the power y under the hairline on the appropriate LL scale, (check the reading rules).

With the slide set to the value of the base "a", we obtain a complete table of values of the function $y = a^x$. Fig. 47 shows such a setting for the function $y = 3.2^x$, in which the cursor hairline is over the value of the exponent $x = 2.5$ and simultaneously its decimal variants.

Examples:

		Read on scale
$3.2^{2.5}$	$= 18.3$	LL3
$3.2^{0.25}$	$= 1.338$	LL2
$3.2^{0.025}$	$= 1.02956$	LL1
$3.2^{0.0025}$	$= 1.002912$	LL0
$3.2^{-2.5}$	$= 0.0546$	LL03
$3.2^{-0.25}$	$= 0.7476$	LL02
$3.2^{-0.025}$	$= 0.97134$	LL01
$3.2^{-0.0025}$	$= 0.997096$	LL00

Reading rules for $y = a^x$

a) When the exponent x is positive, set and read in the same scale group, LL0-LL3 or LL00-LL03, using figuring of like colour.

With negative values of x, it is necessary to switch from one scale group to the other and to set and read in unlike colours.

b) In conformity with the indications given at the right hand end of each scale, read on the adjoining scale with lower value, for each place that the decimal point in the exponent is moved to the left (see example in fig. 47).

c) When the base is set with the right hand slide index, all readings must be taken on the adjoining "higher" value LogLog scale (fig. 49).

With bases $0 < a < 1$, powers for positive exponents are read in scale group LL00-LL03. If the exponent is negative, read the power on LL0-LL3.

Examples:

$0.685^{2.7} = 0.36$ (Fig. 48)

$0.685^{-2.7} = 2.78$

$1.46^{2.7} = 2.78$

$1.46^{-2.7} = 0.36$

Figure 49 shows the examples of fig. 48, but with initial setting using the right hand index of the slide. In this event, the result is not found on the scale in which the base is set, but in the adjacent scale, LL3 or LL03. When the base, as in this example, lies near the middle of the scale, it is often of advantage to work with scale CF. The complete scale range of CF is then available for the setting of the exponent and slide re-setting, as in tabulation, is avoided.

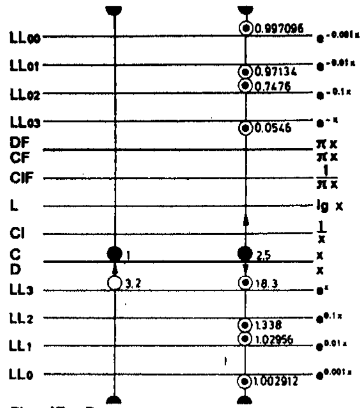

Fig. 47 Powers

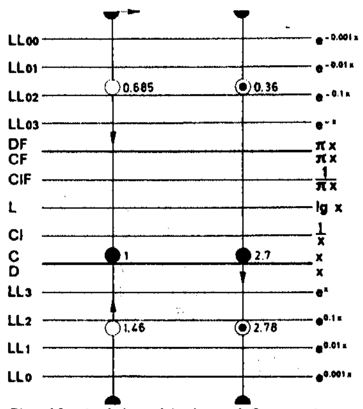

Fig. 48 Left hand index of C over base

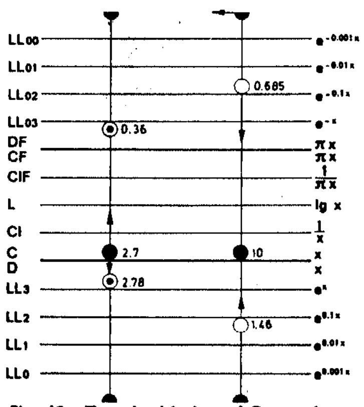

Fig. 49 Terminal index of C over base

If one begins with an easily developed approximation, there is little uncertainty about the magnitude of a power read from the LogLog scales, especially if the several LL scales are recognised as sucessive segments of one continuous exponential scale, interrupted at only one stage, between 0.999 and 1.001. Values in this gap can be found by approximation, using a series, as explained in sections 17.3.2. and 17.3.3.

17.3 Special cases of $y = a^x$

The extent to which the exponent or the base may vary is limited in direct application by the range of the LogLog scale.

17.3.1 $y > 10^5$ and $y < 10^{-5}$

If the power corresponding to a base with a large exponent is outside the range of the LogLog scales, the procedure to be adopted is to express the exponent as a sum and thus obtain the power in factor form.

Example:

$3.14^{19} = 3.14^{6+6+7} = (3.14^6)^2 \times 3.14^7 = 0.955^2 \times 10^6 \times 3.02 \times 10^3 = 2.76 \times 10^9$

Analogous procedure is, of course, appropriate for negative exponents.

17.3.2 $0.999 < y < 1.001$
(ARISTO HyperLog only)

When the exponent is so small that the number raised to a power is less than 1.001 but greater than 0.999, the result cannot be read on the LogLog scales.

The series expansion

$$a^{\pm x} = 1 \pm \frac{x}{1!} \ln a + \frac{x^2}{2!} \ln^2 a \pm \frac{x^3}{3!} \ln^3 a + \ldots$$

provides in such cases an approximate solution in the form:

$$a^{\pm x} \approx 1 \pm x \ln a \text{ for } |x \ln a| \ll 1$$

When index 1 of C is set to the base a on the LogLog scale by means of the cursor, the value of $\log_e a$ is simultaneously set on the D scale (see section 18.4 and 18.6). Multiplication by x is achieved by moving the cursor along the C scale and reading $x \log_e a$ on D. This intermediate value, with 1 added or subtracted, is the required power $a^{\pm x}$. The smaller the exponent, the closer the approximation secured by this method.

The example of fig. 47 can by this means be carried further, thus:

$$3.2^{0.00025} = 1 + 0.0002908 = 1.0002908$$
$$3.2^{-0.00025} = 1 - 0.0002908 = 0.9997092$$

Should the exponent be still further reduced by change in the position of the decimal point, the answer obtained by the method given above will be varied only in respect of the number of zeros or nines immediately following the decimal point. E. g., $3.2^{0.000025} = 1.00002908$.

17.3.3 $0.999 < a < 1.001$
(ARISTO HyperLog only)

When the power $y = a^x$ the base is greater than 0.999 but less than 1.001, an approximation is again of service.

From the series previously quoted, $a^{\pm x} \approx 1 \pm x \ln a$. As a approaches 1, we can write $a = 1 \pm n$ and hence:

$$a^x = (1 \pm n)^x \approx 1 + x \ln (1 \pm n)$$

$$\text{Now } \ln (1 \pm n) = \pm n - \frac{n^2}{2} \pm \frac{n^3}{3} - \ldots \approx \pm n \text{ (for } |n| \ll 1)$$

$$(1 \pm n)^x \approx 1 \pm n\,x \text{ and } (1 \pm n)^{-x} \approx 1 \mp n\,x \text{ (for } |n\,x| \ll 1)$$

If the range of the LogLog scales will not permit setting the base a, scale D can be used as an exponential scale. In this event, note a difference in procedure. In place of $a = 1 \pm n$, we must set the value n. When the initial index 1 of C is brought over n on scale D, the setting is for all practical purposes identical with the setting of $1 \pm n$ on an exponential scale which could be regarded as an imaginary LogLog scale covering the range 1.001 to 1.01 or 0.990 to 0.999. The smaller the value of n, the closer the approximation $\ln (1 \pm n) = \pm n$.

The value of the power is obtained, as usual, by a simple multiplication $n \times x$. To complete the result, the value found thus on D must be added to or subtracted from 1, according to the sign of n. With larger exponents, the power will lie within the range of the LogLog scales and the result can then be read directly from them.

Examples:

$1.00023^{3.7} = (1 + 0.00023)^{3.7} = 1.000851$	Reading on scale D added to 1
$1.00023^{37} = 1.00854$	Reading on scale LL0
$0.99977^{3.7} = (1 - 0.00023)^{3.7} = 0.999149$	Reading on scale D subtracted from 1
$0.99977^{37} = 0.99152$	Reading on scale LL00

17.3.4 0.99 < y < 1.01
(ARISTO HyperboLog only)

When the exponent is so small that the number raised to a power is less than 1.01 but greater than 0.99, the result cannot be read on the LogLog scales.

The series expansion

$$a^{\pm x} = 1 \pm \frac{x}{1!} \ln a + \frac{x^2}{2!} \ln^2 a \pm \frac{x^3}{3!} \ln^3 a + \ldots$$

provides in such cases an approximate solution in the form:

$$a^{\pm x} \approx 1 \pm x \ln a \quad \text{for } |x \ln a| \ll 1$$

When index 1 of C is set to the base a on the LogLog scale by means of the cursor, the value of $\log_e a$ is simultaneously set on the D scale (see section 17.4 and 17.6). Multiplication by x is achieved by moving the cursor along the C scale and reading $x \log_e a$ on D. This intermediate value, with 1 added or subtracted, is the required power $a^{\pm x}$. The smaller the exponent, the closer the approximation secured by this method.

Example:

$$3.2^{0.0025} \approx 1 + 0.0025 \times \ln 3.2$$
$$\approx 1 + 0.002908 = 1.002908$$
$$3.2^{-0.0025} \approx 1 - 0.002908 = 0.997092$$

Should the exponent be still further reduced by change in the position of the decimal point, the answer obtained by the method given above will be varied only in respect of the number of zeros or nines immediately following the decimal point.

E. g. $\quad 3.2^{0.00025} = 1.0002908$

17.3.5 0.99 < a < 1.01
(ARISTO HyperboLog only)

When, in the power $y = a^x$, the base exceeds 0.99 but is less than 1.01, the solution is again obtained by approximation.

In accordance with the series expansion applied to the case reviewed in the preceding paragraph: $a^{\pm x} \approx 1 \pm x \ln a$. Since a, in the present case, is near 1 we can write $a = 1 \pm n$, from which we can further derive:

$$a^x = (1 \pm n)^x \approx 1 + x \ln (1 \pm n)$$

$$\ln (1 \pm n) = \pm n - \frac{n^2}{2} \pm \frac{n^3}{3} - \ldots$$

$$\ln (1 \pm n) \approx \pm n \quad (\text{for } |n| \ll 1)$$
$$(1 \pm n)^x \approx 1 \pm n\,x \quad (\text{for } |nx| \ll 1)$$
$$(1 \pm n)^{-x} \approx 1 \mp n\,x \quad (\text{for } |nx| \ll 1)$$

If the range of the LL scales will not permit setting the base a, scale D can be used as an exponential scale. In this event, note a difference in procedure. In place of $a = 1 \pm n$, we must set the value $|n|$.

When the initial index 1 of scale C is brought over n on scale D, the setting is for all practical purposes identical with the setting of $1 \pm n$ on an exponential scale which could be looked upon as an imaginary LogLog scale covering the range 1.001 to 1.01 or 0.99 to 0.999. The smaller the value of n, the closer the approximation $\ln (1 \pm n) \approx \pm n$.

The value of the power is obtained, as usual, by a simple multiplication n x. To complete the result, the value found on D must be added to 1 or subtracted from 1, according to the sign of n. With large exponents, the power will lie within the range of the LL scales and the result can then be read directly from the LogLog scales.

Examples:	Read on scale
$1.0023^{3.7} = (1 + 0.0023)^{3.7} = 1.00851$	D and add 1
$1.0023^{37} = 1.0888$	LL1
$0.9977^{3.7} = (1 - 0.0023)^{3.7} = 0.99149$	D and deduct from 1
$0.9977^{37} = 0.9184$	LL01

With the cursor hairline aligned over the left index of D, the amount of displacement relative to the line for 1.01 on LL1 provides a good check on the amount of error in the approximative computation. The maximum degree of error will be introduced into the approximation when both setting and reading take place on scale D in substitution for the LogLog scales.

17.3.6 Improving the accuracy
(ARISTO HyperboLog only)

The precision can be improved when the disparity between reading on the D scale and the actual LogLog scale within the range 1.001 to 1.01 is corrected by also applying both the linear and the quadrature term in the series expansion to the previously discussed procedure.

A)	$\ln (1 \pm n) \approx \pm n (1 \mp n/2)$	for settings of the base on D
B)	$e^{\pm x} \approx 1 \pm x (1 \pm x/2)$	for readings taken from D

When the result is obtained from a LogLog scale, only formula A need be applied before making the setting on scale D. If, however, scale D is used exclusively in a computation, corrections have to be applied to the setting as well as to the answer (formula B).

Example:

$1.0023^{3.7} = 1.00854$

For n = 0.0023 substitute the setting
0.0023 (1 − 1/2 × 0.0023) = 0.0023 × 0.99885 = 0.002297 by slide index on scale D.

The operation required to determine the 3.7th power, viz. 1 + 0.002297 × 3.7, gives 1.00850. This reading, because of its taking place on scale D, requires correction by formula B, as follows:

$$0.00850\ (1 + 1/2 \times 0.00850) = 0.00850 \times 1.00425 = 0.00854$$

After adding the "1", the final answer then is 1.00854 (exactly: 1.0085362). The foregoing computation may at first sight appear rather involved and awkward but will actually be found quite simple after some little practice, so that in time the computer will be able to make the corrections by visual estimate.

Corrections of the kind above reviewed are no longer necessary when the base drops below 1.001, because slide rule accuracy will then be equivalent to that obtainable by approximation.

17.4 Powers $y = e^x$

The expression $y = e^x$ provides a special case for "neutral" positioning of the slide, which then establishes the value e = 2.718 as base. Because with this setting, scale D is in a constant relationship with the LogLog scales, it is sufficient to set the cursor to the exponent on scale D in order to read the power on the LL scale. If the cursor is set to 1.489 on scale D, the following powers can at once be read:

$e^{1.489} = 4{,}43$ $\qquad e^{-1.489} = 0.2260$
$e^{0.1489} = 1.1605$ $\qquad e^{-0.1489} = 0.8618$
$e^{0.01489} = 1.015$ $\qquad e^{-0.01489} = 0.98523$
$e^{0.001489} = 1.001489$ $\qquad e^{-0.001489} = 0.998513$

Further variations can be determined by using the approximation $e^{\pm x} \approx 1 \pm x$

$$e^{0.0001489} = 1.0001489$$

17.5 Roots $a = \sqrt[x]{y}$

Evolution, with given radicands, can be carried through directly with the exponential scales. The extraction of roots, the converse of raising to a power, follows from division using the LogLog scales and fundamental scale C. If the example of a power, $3.2^{2.5} = 18.3$, given in chapter 17.2, is reversed, it will be seen that by working in the contrary sequence we can read $\sqrt[2.5]{18.3} = 3.2$.

Procedure:

a) Set the radicand y on the appropriate LL scale over the radix x on the scale C.

b) Read the value of the root under the initial or final index of the scale C on the appropriate LL scale.

The rules for reading the result, given in chapter 17.2, are here applicable. It is to be noted that when a reading is to be taken under the right hand index, reference must be made to the next lower LL scale.

Examples:

$\sqrt[0.77]{21} = 52.1 \qquad \dfrac{1}{\sqrt[0.77]{21}} = 0.0192$

$\sqrt[7.7]{21} = 1.485 \qquad \dfrac{1}{\sqrt[7.7]{21}} = 0.6734$

$\sqrt[77]{21} = 1.0403 \qquad \dfrac{1}{\sqrt[77]{21}} = 0.96122$

$\sqrt[770]{21} = 1.00396 \qquad \dfrac{1}{\sqrt[770]{21}} = 0.99605$

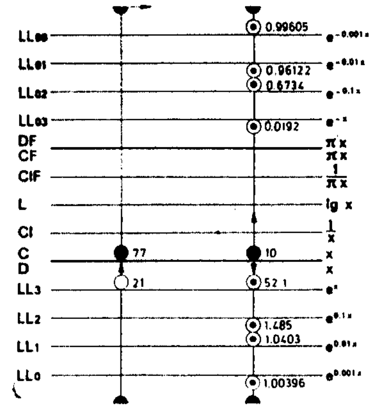

Fig. 50 Roots

The extraction of roots is more easily understood if the radix is expressed as an exponent. The exponent can then be set on scale CI or, if the base is e, on scale DI.

In the following example the cursor is set to 3.5 on DI and the root read on LL2 and LL02.

$$\sqrt[3.5]{e} = e^{+\frac{1}{3.5}} = 1.3307 \text{ in LL2} \qquad \sqrt[\frac{1}{3.5}]{e} = e^{-\frac{1}{3.5}} = 0.7514 \text{ in LL02}$$

17.6 Logarithms

17.6.1 Logarithms to any base

Required logarithms to any base can be found with the LogLog scales. By reversing the process of raising a number to a power, we obtain its logarithm as is seen immediately if we write:

$$y = a^x, \qquad x = \log_a y \text{ (read: logarithm of y to base a).}$$

The finding of a logarithm is thus identical with the problem of a power for which the exponent is required.

Procedure:

a) Set cursor to base a on the appropriate LL scale.

b) Bring the initial or final index of the slide to the cursor hairline.

c) Set the value of y, with the cursor, on the LL scale.

d) Read the logarithm on scale C under the hairline.

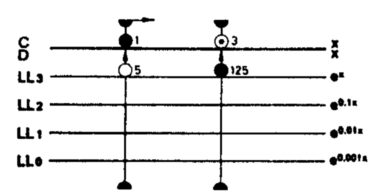

Fig. 51 $\log_5 125 = 3.0$

The position of the decimal point can be determined from the relationship: $\log_a a = 1$.

With the left hand index of the slide over the base a, all values to the right of the value a on scale C are greater than 1 and all values to the left of a on scale C are less than 1.

Reading rules:

a) Passing from one LogLog scale to the adjacent scale, in sequence LL3, LL2, LL1, or LL03, LL02, LL01 results in a shift of the decimal point in the logarithm by one place to the left. A change of scale in the opposite direction calls for a shift of the decimal point to the right.

b) The logarithms will be positive (negative) when their antilogs and bases are set on like (unlike) coloured LogLog scales.

Examples for practice: $\log_2 16 = 4.0$

$\log_2 1.02 = 0.02857$

$\log_2 0.25 = -2$

17.6.2 Decadal logarithms

If the index 1 of scale C is set to base 10 on LL3, then for any number set on an LL scale, the decadal logarithm can be read on scale C (fig. 52).

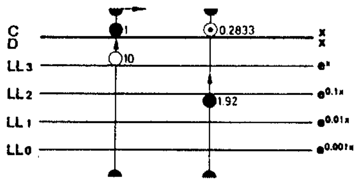

Fig. 52 lg 1.92 = 0.2833

The frequently required decadal logarithms can also be found from the usual mantissa scale L on the slide, if the antilog is set on scale C. Scale L gives only the mantissa and the characteristic must be added in accordance with the rule "number of places minus 1", as when a table of logarithms is used. For every plain number (antilog) on scale C, the logarithm is directly available on scale L and conversely, given the logarithm, the antilog can be read directly from scale C (fig. 53).

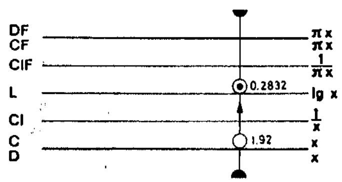

Fig. 53 lg 1.92 = 0.283 with L/C

If scale L is used, it is only necessary to move the cursor and thus the finding of a decadal logarithm is more simple than when the LogLog scales are used. However, within the range of scale LL1, the LogLog scale gives greater precision in reading:

Examples: lg 1.03 = 0.01283 using scale LL1

lg 1.03 = 0.013 using scale L

Examples for practice:

$\log_{10} 50 = 1.699$

$\log_{10} 2 = 0.3010$

$\log_{10} 1.03 = 0.01283$

$\log_{10} 0.015 = -1.824$

$\log_{10} 0.5 = -0.3010$

$\log_{10} 0.1 = -1$

$\log_{10} 6 = 0.778$

$\log_{10} 1.14 = 0.0569$

$\log_{10} 1.015 = 0.00647$

When setting with the right hand index of scale C, all results lie on the left of the base and are therefore < 1, e. g. $\log_{10} 9 = 0.954$. Logarithms of numbers < 1 are negative.

17.6.3 Natural logarithms

Logarithms to base "e" are simply found by transfer from the fundamental scale D to the LogLog scale (fig. 54).

Examples for practice:

ln 4.357 = 1.475
ln 0.622 = −0.475
ln 0.05 = −2.994

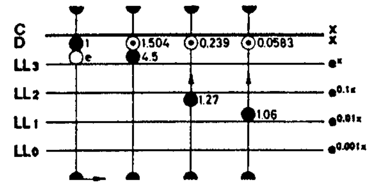

Fig. 54 ln 4.5 = 1.504
ln 1.27 = 0.239
ln 1.06 = 0.0583

18. Further applications of the LogLog scales

Up to this point, we have used only the slide scale C in conjunction with the LogLog scales, in order to demonstrate the essential relationships. Other slide scales can, of course, be used. Their functions, in association with scale C, have already been explained in earlier sections e. g., with scale B, a power $a^{\sqrt{x}}$ can be set. Also, the slide scale of sines, S, is of immediate practical value in calculating $e^{\sin x}$. Reciprocals, too, offer further possibilities in logarithmic computation. Scale CF can be used in place of scale C, in conjunction with the LL scales, to reduce slide re-setting when tabulating, should the base lie near the middle of the scale.

18.1 Solving proportions with the LogLog scales

If the index 1 of scale C is set to a base "a" on a LogLog scale, the powers to any exponent and also the logarithms of any number to this base can be read. A base a set on a LogLog scale can thus be regarded as a term in a proportion.

18.1.1 $y_1 = a^n \qquad y_2 = a^m$

$\log y_1 = n \times \log a$

$\log y_2 = m \times \log a$

$$\frac{\log a}{1} = \frac{\log y_1}{n} = \frac{\log y_2}{m} \text{ or}$$

$$\frac{\ln a}{1} = \frac{\ln y_1}{n} = \frac{\ln y_2}{m}$$

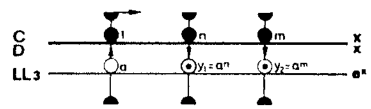

Fig. 55

If three terms of a proportion are known, the fourth proportional can be calculated and with the initial setting, a number of terms in the same ratio can be found. Here again is seen the advantage of the principle of proportionality, a method of calculation for which the slide rule is particularly suitable, as examples will show.

18.1.2

$$y = a^{\frac{m}{n}} \rightarrow \log y = \frac{m}{n} \log a$$

$$\frac{\log y}{m} = \frac{\log a}{n}$$

$$y = 4.3^{\frac{6.8}{2.7}} \rightarrow \frac{\log y}{6.8} = \frac{\log 4.3}{2.7}$$

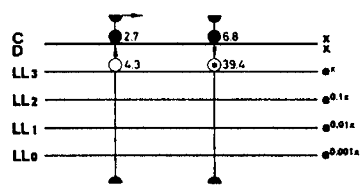

Fig. 56 $\frac{\log 4.3}{2.7} = \frac{\log 39.4}{6.8}$

After setting 4.3 on LL3 opposite 2.7 on scale C, the result 39.4 will be found on scale LL3 under 6.8 of C. Modifications of this problem are, of course, solved analogously:

$$y = \sqrt[2.7]{4.3^{6.8}} \quad \text{or} \quad y^{2.7} = 4.3^{6.8}$$

18.1.3

Many natural laws can be expressed in proportional form, if the change in one variable is proportional to the difference in the logarithm of the other variable, i. e., if we can write:

$$\log y_2 - \log y_1 = \text{const}\,(x_2 - x_1)$$

Because $\log a - \log b = \log \frac{a}{b}$

we may write: $\log \frac{y_2}{y_1} = \text{const.}\,(x_2 - x_1)$.

A change from x_1 to x_2, by an amount i, results in a change of y_1 to y_2. If we denote the ratio $\frac{y_2}{y_1}$ by r, representing the residue of the initial quantity, the above equation can be written:

$$\frac{\log r}{i} = \text{const.} = \frac{\log r_1}{i_1} = \frac{\log r_2}{i_2} = \ldots$$

Example: Radioactive decay

The decomposition rate of a substance is known to be 40% in 30 days, i. e., the residue is then 60%. After how many days will the residue be 20%?

Here $i_1 = 30$, $r_1 = 0.6$, $r_2 = 0.2$

$$\frac{\log 0.6}{30} = \frac{\log 0.2}{x}$$

whence $x = 94.5$ days.

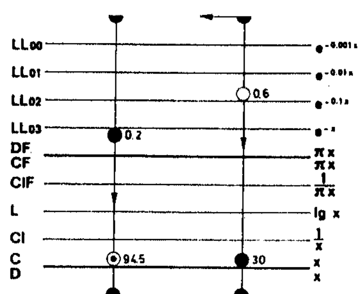

Fig. 57

18.1.4

If a logarithm is to be multiplied by a constant factor, the constant is set on C, over the base of the logarithm on the LogLog scale. A tabulating position for the multiplication is at once set up.

For $x = c \log_a y$, write in proportional form:

$$\frac{x}{\log_a y} = \frac{c}{1} = \frac{c}{\log_a a}$$

$2 \times \log_{10} 100 = 4$

$2 \times \log_{10} 1.8 = 0.511$

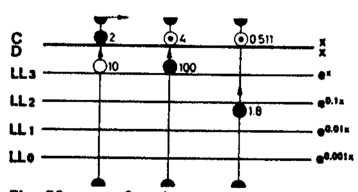

Fig. 58 $x = 2 \times \lg y$

As shown in fig. 58, all logarithms to base 10 can be multiplied by the constant 2. The process applies also to the LL0 scale group, with logarithms of numbers < 1. In physics and communication technology it is often necessary to express a given amplitude ratio in decibel (dB) notation.

$$\text{dB} \triangleq 20 \lg \frac{A_1}{A_2}$$

Examples:

20 dB = 20 lg 10
40 dB = 20 lg 100
5.11 dB = 20 lg 1.8

19. The hyperbolic functions

The scales Sh1, Sh2, Ch and Th, like trigonometrical scales, link angles to function values read on scale D. For any argument set on Sh, Ch or Th, the corresponding function value is at once readable on scale D.

For the hyperbolic functions the respective arguments are given in radians and not in degrees. Conversion of degrees to radians and vice-versa can be performed by the method given in chap. 15.2, the respective values appear directly under the cursor hairline on scales C and ST.

Multiplication and division with scales Sh1, Sh2, Ch and Th is analogous in procedure to that employed when the trigonometric scales are involved so that expressions of the form cos y × sinh x etc. can be worked out.

19.1 Scales Sh1, Sh2

For any argument x, between 0.1 and 0.881, set on scale Sh1, function values of sinh x between 0.1 and 1.0 are available on D. The additional scale Sh2 extends this facility for arguments x between 0.85 and 3, with the corresponding values of sinh x between 1 and 10.

Note: For $x > 3$, $\sinh x \approx \frac{e^x}{2}$

and $x < 0.1$, $\sinh x \approx x$

Examples:

1. sinh 0.349 = 0.356
2. sinh 0.885 = 1.005
3. sinh 1.742 = 2.77

Naturally, the converse applies — for a given function value the argument, in radian measure, can be read.

For sinh x = 2.77, the converse is

x = arc sinh 2.77 = 1.742.

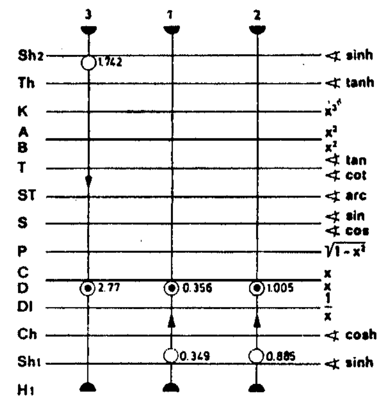

Fig. 59 Hyperbolic sine

19.1.1 Cosh x
(ARISTO HyperboLog only)

The computation of the hyperbolic cosine is somewhat more complicated. Two methods of solution may be used, viz.:

$$\cosh x = \frac{\sinh x}{\tanh x} \quad \text{or} \quad \cosh x = \sqrt{\sinh^2 x + 1}$$

Fig. 60 illustrates the solution of cosh 0.437 by performing the division sinh x ÷ tanh x. The computation begins with setting the index of the slide to the argument on scale Th with the aid of the cursor. The value of cosh x is then read on scale C directly over the same argument on the Sh scale. The value so obtained represents the difference between the two scale values pertaining to 0.437 as measured along scale C.

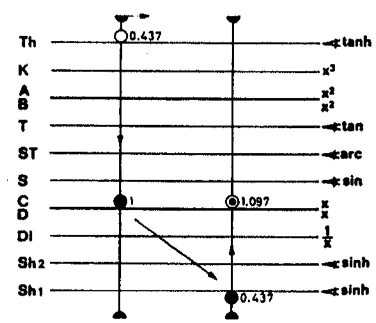

Fig. 60 cosh 0.437 = 1.097

$$\cosh 0.437 = \frac{\sinh 0.437}{\tanh 0.437} = 1.907$$

or, better, in the proportion form:

$$\frac{\tanh 0.437}{1} = \frac{\sinh 0.437}{\cosh 0.437}$$

$$\cosh 1.5 = \sqrt{\sinh^2 1.5 + 1} = 2.352$$

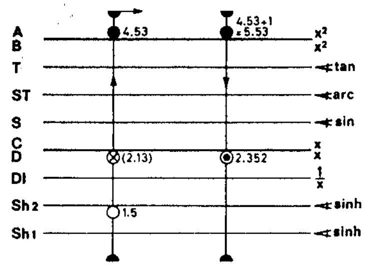

Fig. 61 cosh 1.5 = 2.352

The solution shown in fig. 61 takes the following course: Set the cursor to the argument x on scale Sh2 and read the value of $\sinh^2 x$ opposite this setting on scale A. After adding "1" to the value so obtained, shift the cursor to this new value $\sinh^2 x + 1$ on scale A. Scale D then supplies the answer under the cursor hairline. This form of solution is quite simple but requires the computer's strict attention to the correct placing of the decimal point in the square, so as to make sure that the cursor line is set to the correct sum and within the appropriate section of scale A. The advantage of this method lies in the fact that the course of the computation may be reversed when the argument has to be determined from the function.

Examples:

$\cosh 0.2 = 1.02$ $\cosh 1.0 = 1.543$ $\text{arc}\cosh 2.5 = \text{arc}\sinh \sqrt{2.5^2 - 1} = 1.567$

When $x < 0.1$ write $\cosh x \approx 1$. For $x > 3$ apply $\cosh x \approx \frac{e^x}{2}$

19.2 Scale Ch
(ARISTO HyperLog only)

For any argument $0 < x < 3$ set on Ch we have at once the function value cosh x, between 1 and 10, on scale D.

Note: For $x < 0.1$, $\cosh x \approx 1$

$x > 3$, $\cosh x \approx \frac{e^x}{2}$

$\approx \sinh x$

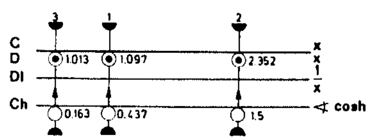

Fig. 62 Hyperbolic cosine

Examples:

1. $\cosh 0.437 = 1.097$
2. $\cosh 0.163 = 1.013$
3. $\cosh 1.5 \quad = 2.352$

Scales H1 and H2, in conjunction with scales Sh1 and Sh2, provide further possibilities for the determination of hyperbolic cosines, by use of the identity

$$\cosh x = \sqrt{1 + \sinh^2 x}$$

For any argument set on scale Sh we have the function value on D and for any value x on D, the value $\sqrt{1 + x^2}$ can be read on H. Thus, setting any argument x on scale Sh has, opposite to it on scale H, cosh x.

Opposite the range $0.1 < x < 1.0$ on D we have $1.005 < \sqrt{1 + x^2} < 1.41$ on H1 and against $1.0 < x < 10$ on D, $1.41 < \sqrt{1 + x^2} < 10.06$ on H2.

Reference to section 19.1 shows that, for hyperbolic cosines, scales Sh1, H1 and Sh2, H2 work in pairs.

If settings on the left hand portion of scale Ch are uncertain, scales Sh1, H1 provide greater accuracy. In practice, we have a correspondence resembling that between scales S and P.

Fig. 63 shows examples for comparison with those of fig. 62.

$\cosh 0.437 = 1.0971$

$\cosh 1.5 \quad = 2.352$

$\cosh 0.163 = 1.0133$

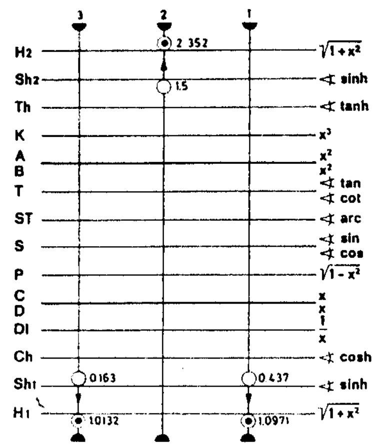

Fig. 63 Hyperbolic cosine

19.3 Scale Th

This scale provides for arguments x, from 0.1 to 3, in association with scale D, corresponding function values of tanh x, 0.1 to 0.995.

For $x > 3$, $\tanh x \approx 1 - 2\,e^{-2x} \approx 1$
for $x < 0.1$, $\tanh x \approx x$

Examples:
$\tanh 0.257 = 0.251$
$\tanh 1.614 = 0.924$

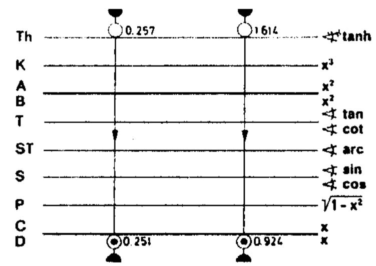

Fig. 64 Hyperbolic tangent

The function values of coth x can be found from $\coth x = \frac{1}{\tanh x}$ on scale D, if x is set on scale Th.

Example:

$\tanh 0.549 = 0.500$ — For $x < 0.1$, $\coth x \approx \frac{1}{x}$

$\coth 0.549 = 2.000$ — For $x > 3$, $\coth x \approx 1$

19.4 Fundamental hyperbolic formulae

It is often convenient to have ready to hand a list of the fundamental hyperbolic formulae:

$\sinh x = {}^1/_2\,(e^{+x} - e^{-x})$ — $\sinh x + \cosh x = e^x$

$\cosh x = {}^1/_2\,(e^{+x} + e^{-x})$ — $-\sinh x + \cosh x = e^{-x}$

$\tanh x = \frac{\sinh x}{\cosh x}$ — $\tanh x = \frac{1 - e^{-2x}}{1 + e^{-2x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$

$\tanh x \times \coth x = 1$ — $\cosh^2 x - \sinh^2 x = 1$

20. Hyperbolic functions with a complex argument

In the literature of mathematics it is shown how the following formulae for circular trigonometric and hyperbolic functions, with complex arguments, are derived.

(a) $\sinh(x \pm j\,y) = \sinh x \times \cos y \pm j \cosh x \times \sin y$

(b) $\cosh(x \pm j\,y) = \cosh x \times \cos y \pm j \sinh x \times \sin y$

(c) $\sin(x \pm j\,y) = \sin x \times \cosh y \pm j \cos x \times \sinh y$

(d) $\cos(x \pm j\,y) = \cos x \times \cosh y \pm j \sin x \times \sinh y$

(e) $\tanh(x \pm j\,y) = \dfrac{\tanh x \pm j \tan y}{1 \pm j \tanh x \times \tan y}$

(f) $\tan(x \pm j\,y) = \dfrac{\tan x \pm j \tanh y}{1 \mp j \tan x \times \tanh y}$

Observe that in these formulas the arguments of the functions concerned may be either radians or degrees.

By use of formulas (a) to (d) the answers are at first obtained in the complex notation form $a + j\,b$, from which the vector equivalent r/φ can be computed in the customary manner as shown in the examples given in chap. 16.1.

Examples:

$$\sinh(0.25 + j12.7^\circ) = \sinh 0.25 \times \cos 12.7^\circ + j \cosh 0.25 \times \sin 12.7^\circ$$
$$= 0.2526 \times 0.976 + j\,1.031 \times 0.2198$$
$$= 0.2464 + j\,0.2267$$
$$= 0.335\underline{/42.6^\circ}$$

$$\sin(1.05 + j0.61) = \sin 60.2^\circ \times \cosh 0.61 + j \cos 60.2^\circ \times \sinh 0.61$$
$$= 1.035 + j\,0.322 = 1.083\underline{/17.3^\circ}$$

Intermediate conversion: 1.05 radians = 60.2°

Although the indicated multiplications are performed without stopping to read the several values of the functions themselves, the computation is rather tedious, especially in view of the inconvenience in computing the cosh. Therefore the following method is presented to show a short cut to the solution of these problems.

20.1 Sinh (x + j y)

The identity $\sinh(x + j\,y) = \sinh x \times \cos y + j \cosh x \times \sin y$ can be written in the familiar vector form as $a = \sinh x \times \cos y$ and $b = \cosh x \times \sin y$. See fig. 65.

From figure 65, compute φ from:

$$\tan\varphi = \frac{\cosh x \times \sin y}{\sinh x \times \cos y} = \frac{\cosh x}{\sinh x} \times \frac{\sin y}{\cos y} = \frac{\tan y}{\tanh x}$$

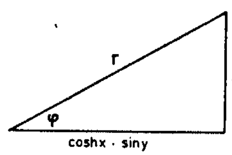

Fig. 65

This new formula makes possible a speedier calculation for φ and r, since if φ is known from $\tan\varphi$,

$$r = \frac{\sinh x \times \cos y}{\cos\varphi}$$

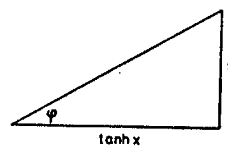

Fig. 66

This latter equation can be seen from fig. 66. Squaring the values of the sides provides a check on the work, since $r^2 = \sinh^2 x + \sin^2 y$. Whilst the division $\tan\varphi = \dfrac{\tan y}{\tanh x}$ is a fairly simple problem, the shortest method may not be

immediately apparent, since, according to the magnitude of the arguments φ and y, several setting methods may be used. The best recourse, to avoid reading errors, is to make an approximation, using rounded-off values.

Example: sinh (0.25 + j 12.7°)

It can be seen at a glance that tan 12.7° on T is located to the left of tanh 0.25 on scale Th and is therefore the smaller one of the two values. Hence, too, $\tan\varphi < 1$ and $\varphi < 45°$.

$$\tan\varphi = \frac{\tan 12.7^\circ}{\tanh 0.25} \text{ or, written in proportion form } \frac{\tanh 0.25}{\tan 12.7^\circ} = \frac{1}{\tan\varphi}$$

The latter form is the best suited for slide rule work. We only have to bring 0.25 on Th into coincidence with 12.7° on scale T in order to read φ over the right index of the D scale on T.

$$\tan\varphi = \frac{\tan 12.7^\circ}{\tanh 0.25} = 42.6^\circ$$

Approximation:

$$\tan\varphi \approx \frac{0.2}{0.25} = 0.8,\ \varphi \approx 40^\circ$$

$$r = \frac{\sinh 0.25}{\cos 42.6^\circ} \times \cos 12.7^\circ = 0.335$$

$$r = \sqrt{\sinh^2 0.25 + \sin^2 12.7^\circ}$$

$$r = \sqrt{0.0484 + 0.0639} = 0.335$$

Check:

Sinh (0.25 + j 12.7°) = $0.335\underline{/42.6^\circ}$

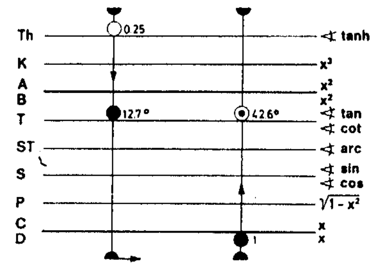

Fig. 67 φ = 42.6°

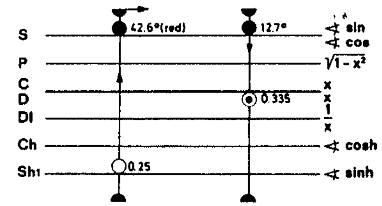

Fig. 68 r = 0.335

To enable this short-cut method to be used for all combinations of angles, setting and reading sequences are gathered together in the following table. Whoever meets such problems regularly will quickly master the method and save much time. For those who rarely meet such problems or do not have the table at hand, it is probably best and safer to determine the values of the functions tanh x and tan y as a first step, before attempting the division.

Table: calculation of $\dfrac{\tan y}{\tanh x}$

y	x	First setting y on	Cursor setting over	Reading for φ on	φ
5.5°—45°	tanh x > tan y	T under x on Th	right index of D	T	< 45°
5.5°—45°	tanh x < tan y	T under x on Th	right index of C, then slide closed	T (red)	> 45°
< 5.5°	tanh x < 10 tan y	ST under x on Th	left index of D	T	> 5.5°
< 5.5°	tanh x > 10 tan y	ST under x on Th	right index of D	ST	< 5.5°
> 45°	10 tanh x > tan y	T (red over) right index of D	x on Th	T (red)	> 45°
> 45°	10 tanh x < tan y	T (red) over left index of D	x on Th	ST	> 45°

Evaluation of the hypotenuse

$$r = \frac{\sinh x}{\cos \varphi} \times \cos y$$

is simple if it is remembered that all cosines must be set on scale S, using the red figures.

Examples, using the table:

1.

sinh (0.361 + j 11.8°) = 0.422/31.12°
tanh 0.361 > tan 11.8°
$\varphi = 31.12°$

$$r = \frac{\sinh 0.361}{\cos 31.12°} \times \cos 11.8°$$

r = 0.422

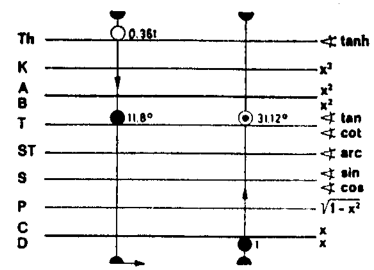

Fig. 69 φ = 31.12°

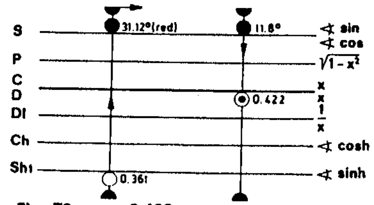

Fig. 70 r = 0.422

2.

sinh (0.38 + j 32°) = 0.657/59.87°
tanh 0.38 > tan 32°
$\varphi = 59.87°$

Set tanh 0.38 and tan 32° opposite each other (fig. 71) and move cursor to the slide index to read on scale DI the value tan φ = 1.725. Without moving the cursor, bring next the slide to the "neutral" position, when φ = 59.87° can be read on scale T.

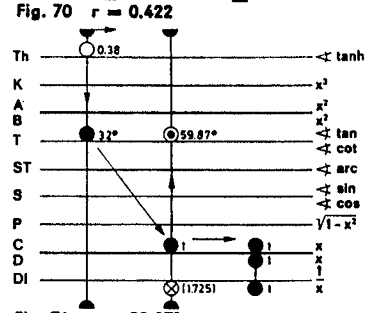

Fig. 71 φ = 59.87°

3.

sinh (0.262 + j 4.52°) = 0.2764/17.13°
tanh 0.262 < 10 × tan 4.52°
$\varphi = 17.13°$

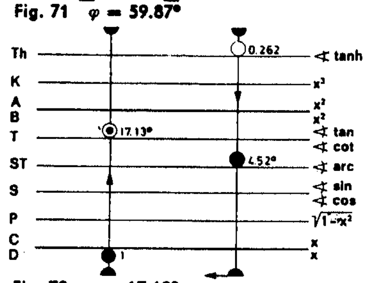

Fig. 72 φ = 17.13°

4.

sinh (1.13 + j 3.8°) = 1.388/4.68°
tanh 1.13 > 10 tan 3.8°
$\varphi = 4.68°$

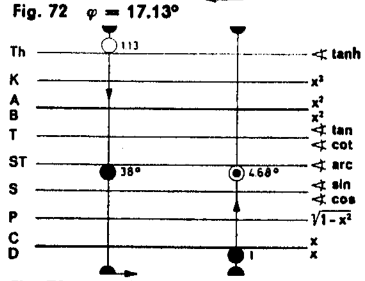

Fig. 73 φ = 4.68°

5.

sinh (0.195 + j 51.1°) = 0.8025/81.17°
10 × tanh 0.195 > tan 51.1°
$\varphi = 81.17°$

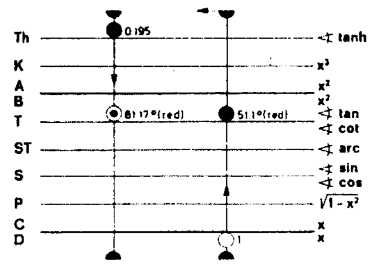

Fig. 74 φ = 81.17°

6.

sinh (0.274 + j 73.5°) = 0.997/85.47°
10 × tanh 0.274 < tan 73.5°
$\varphi = 85.47°$

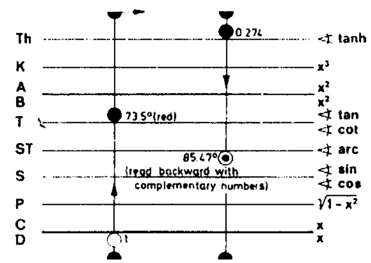

Fig. 75 φ = 85.47°

$$r = \frac{\sinh 0.274}{\cos 85.47°} \times \cos 73.5°$$

r = 0.997

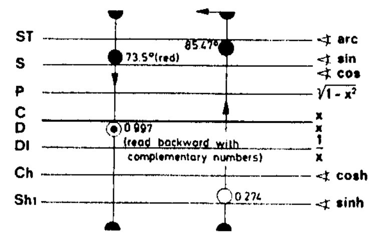

Fig. 76 r = 0.997

20.2 cosh (x + j y)

In a manner analogous to the previous description the hyperbolic cosine of a complex argument can also be computed.

cosh (x + j y) = (cosh x × cos y) + j (sinh x × sin y)

(a) $\tan \varphi = \tan y \times \tanh x$

(b) $r = \frac{\sinh x \times \sin y}{\sin \varphi}$

(c) $r^2 = \sinh^2 x + \cos^2 y$

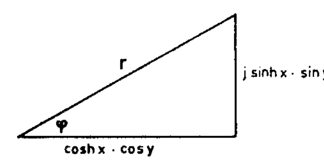

Fig. 77

Equation (c) again gives a means for finding r without knowing the angle φ by visualizing a rightangled triangle with the sides sinh x and cos y as well as the hypotenuse r. However, more frequently r and φ are unknown. The angle φ is then computed from Eq. (a) and the hypotenuse r from Eq. (b). While it is quite easy to solve for r, the double use of the tan function in Eq. (a) needs some rational thinking, particularly when angles > 45° are concerned.

In problems of this kind, also, it is preferable to solve occasional problems by following the elementary but safer course. Those users who have to make these computations as a matter of routine, may refer to the summary printed below, which indicates the sequence of settings to be used for the fastest and most accurate solutions of problems involving the expression cosh (x + j y).

y	x	First Setting	Second Setting	Read φ on	φ
< 45°	0.1 < x < 3.0	y on T over index of D	Cursor to x on Th	T or ST resp. (black)	< 45°
A rough approximation will settle whether the angle must be read on T or ST:		a) 0.1 < tanh x × tan y < 1 on T b) 0.01 < tanh x × tan y < 0.1 on ST c) tanh x × tan y < 0.01 on ST			< 45° < 5.7° < 0.57°
> 45°	cot y > tanh x	Rule closed, cursor over y in T (red)	Right slide index under cursor line	Cursor to x on Th. Answer below on T (black)	< 45°
Setting the cursor to y on T gives cot y on C					
> 45°	cot y < tanh x	y on T (red) under x on Th	Cursor over index of D	T (red)	> 45°

Examples, using the table:

1 a.

cosh (0.523 + j 38.6°) = 0.954/20.97°

tanh 0.523 × tan 38.6° ≈ 0.4,

$\varphi = 20.97°$

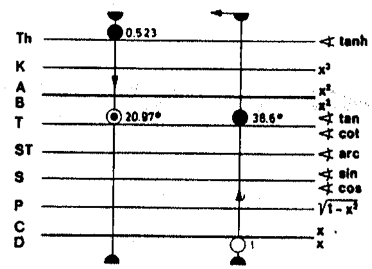

Fig. 78 φ = 20.97°

$$r = \frac{\sinh 0.523}{\sin 20.97°} \times \sin 38.6°$$

r = 0.954

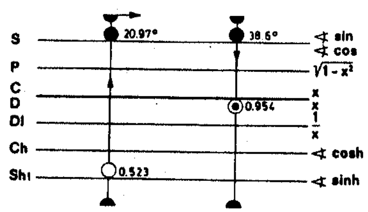

Fig. 79 r = 0.954

1 b.

cosh (0.261 + j 20.06°) = 0.976/5.32°

tanh 0.261 × tan 20.06° ≈ 0.09

$\varphi = 5.32°$

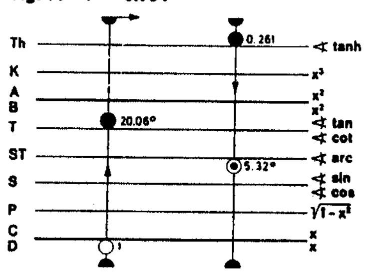

Fig. 80 φ = 5.32°

$$r = \frac{\sinh 0.261}{\sin 5.32°} \times \sin 20.06°$$

r = 0.976

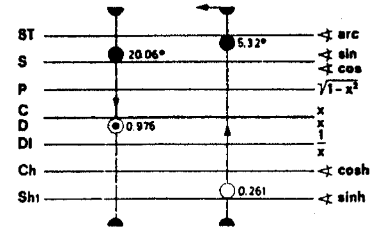

Fig. 81 r = 0.976

1 c.

cosh (0.183 + j 2.31°) = 1.020/0.417°

tanh 0.183 × tan 2.31° ≈ 0.008

$\varphi = 0.417°$

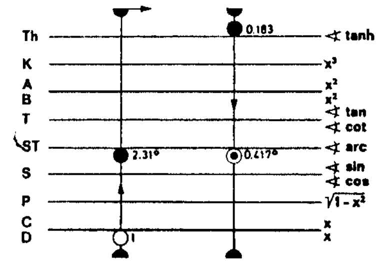

Fig. 82 φ = 0.417°

2.

cosh (0.525 + j 52.4°) = 0.821/32°

cot 52.4° > tanh 0.525,

$\varphi = 32°$

Referring to fig. 83, with the slide in the "neutral" position, tan 52.4° is set on scale T with the cursor. The slide index is then brought under the cursor hair line.

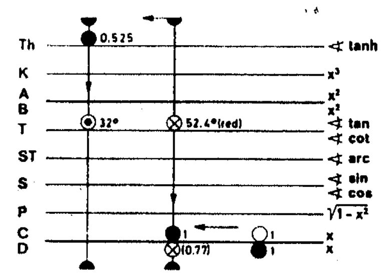

Fig. 83 φ = 32°

cosh (0.318 + j 79.5°) = 0.371/58.92°

cot 79.5° < tanh 0.318

$\varphi = 58.92°$

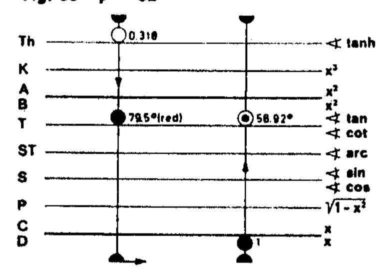

Fig. 84 φ = 58.92°

$$r = \frac{\sinh 0.318}{\sin 58.92°} \times \sin 79.5°$$

r = 0.371

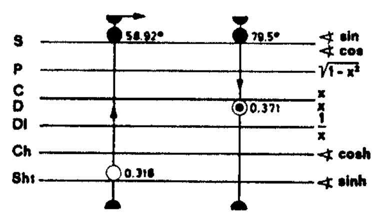

Fig. 85 r = 0.371

20.3 tanh (x ± j y)

This expression can be calculated in two ways:

(a) $\tanh(x \pm j\,y) = \dfrac{\sinh(x \pm j\,y)}{\cosh(x \pm j\,y)}$ (b) $\tanh(x \pm j\,y) = \dfrac{\tanh x \pm j \tan y}{1 \pm j \tanh x \times \tan y}$

Which of the two solutions offers the greater convenience is a matter of individual preference. Users with sound practice in the computation of sinh (x + j y) and cosh (x + j y) will probably adopt formula (a) as the speedier method.

Example: $\tanh(0.25 + j\,12.7^\circ)$

$$\text{Formula (a) gives: } \frac{0.335/\underline{42.62^\circ}}{1.008/\underline{3.16^\circ}} = 0.333/\underline{39.46^\circ}$$

$$\text{Formula (b) gives: } \frac{\tanh 0.25 + j \tan 12.7^\circ}{1 + j \tanh 0.25 \times \tan 12.7^\circ} = \frac{0.245 + j\,0.225}{1 + j\,0.245 \times 0.225}$$

$$= \frac{0.245 + j\,0.225}{1 + j\,0.0552} = \frac{0.333/\underline{42.62^\circ}}{1.00/\underline{3.16^\circ}} = 0.333/\underline{39.46^\circ}$$

21. Trigonometrical functions with complex argument

1. For the calculation of sin (x + j y), the undermentioned formulae are applicable:

$$\tan\varphi = \frac{\tanh y}{\tan x} \qquad r = \frac{\sinh y \times \cos x}{\sin\varphi}$$

Example: $\sin(0.52 + j\,0.24)$

$$\tan\varphi = \frac{\tanh 0.24}{\tan 29.8^\circ} = 0.411$$

$$r = \frac{\sinh 0.24 \times \cos 29.8^\circ}{\sin 22.4^\circ}$$

$$\varphi = 0.552$$

2. Corresponding formulae for cos (x + j y) are:

$$\tan\varphi = \tanh y \times \tan x \qquad r = \frac{\sinh y \times \sin x}{\sin\varphi}$$

Example: $\cos(0.52 + j\,0.24)$

$$\tan\varphi = \tanh 0.24 \times \tan 29.8^\circ = 0.135$$

$$\varphi = 7.7^\circ$$

$$r = \frac{\sinh 0.24 \times \sin 29.8^\circ}{\sin 7.7^\circ}$$

$$= 0.899$$

3. When calculating tan (x + j y), the formula (f) of section 20 will apply, but solution by the division

$$\tan(x + j\,y) = \frac{\sin(x + j\,y)}{\cos(x + j\,y)}$$

will be speedier.

22. Conversion of examples of sections 20 and 21

When a problem is expressed in the reverse order and consists of computing the complex argument x + j y when its hyperbolic function is given, the procedure is inevitably somewhat more intricate. The required operations will be explained step-by-step for the hyperbolic sine functions.

22.1 ar sinh $r/\underline{\varphi} = x + j\,y$

To compute the vectorial expression $r/\underline{\varphi}$ for sinh (x + j y), the components a and b must be found. This presents little difficulty — see sections 16 and 16.1.

$$\sinh(x + j\,y) = r/\underline{\varphi} = r\cos\varphi + j\,r\sin\varphi = a + j\,b$$

In this, x and y are functions of a and b.

From the equation $\sinh(x + j\,y) = \sin x \times \cos y + j \cosh x \times \sin y = a + j\,b$ two further equations, with two unknowns, can be derived and solved for x and y:

$$a^2 + (1 + b)^2 = M^2 = (\cosh x + \sin y)^2$$
$$a^2 + (1 - b)^2 = N^2 = (\cosh x - \sin y)^2$$

$$\text{whence } 2\cosh x = M + N$$
$$2\sin y = M - N$$

y is then found from $\sin y = \dfrac{M - N}{2}$ and x from the equation $\sinh x = \dfrac{a}{\cos y}$

or, alternatively, the relationship $\cosh x = \dfrac{(M + N)}{2}$.

The foregoing equations show that M and N can be regarded as the hypotenuse of two right triangles of which the sides are, respectively, a and (1 + b) and a and (1 − b). These triangles can then be solved by the method of sections 13 and 16, the angles having, in this case, no significance.

Example:

$\sinh(x + j\,y) = 0.422/\underline{31.12^\circ}$ (cf. example 1 of page 41).

Required: x and y

$0.422/\underline{31.12^\circ} = a + j\,b = 0.361 + j\,0.218$

The right triangle with a = 0.361 and (1 + b) = 1.218 gives M = 1.270

The right triangle with a = 0.361 and (1 − b) = 0.782 gives N = 0.861

$M - N = 0.409$ $\qquad M + N = 2.131 = 2\cosh x$

$^1/_2\,(M - N) = 0.2045 = \sin y$ $\qquad \cosh x = 1.0655$

$y = 11.8^\circ$ $\qquad x = 0.360$

Under 1.0655 on scale D is x = 0.360 on scale Ch

Over 1.0655 on scale H1 is x = 0.360 on Sh1

22.2 ar cosh $r/\underline{\varphi} = x + j\,y$

The solution takes the same course as in the preceding problem except that the formulas are adapted to the hyperbolic cosine.

$$M^2 = (1 + a)^2 + b^2 \qquad \cos y = {}^1/_2\,(M - N)$$

$$N^2 = (1 - a)^2 + b^2 \qquad \sinh x = \frac{b}{\sin y}$$

Example: $\cosh(x + j\,y) = 0.954/\underline{20.97^\circ}$ (Compare example 1 a. on page 44)

$a + j\,b = 0.892 + j\,0.3413$

triangle with 1 + a = 1.892 and b = 0.3413 gives M = 1.922

triangle with 1 − a = 0.108 and b = 0.3413 gives N = 38.6°

$M - N = 1.564$ $\quad {}^1/_2\,(M - N) = 0.782 = \cos y$ $\qquad y = 0.358$

$\sinh x = \dfrac{0.3413}{\sin 38.6^\circ}$ $\qquad x = 0.523$

$0.954/\underline{20.97^\circ} = \cosh(0.523 + j\,38.6^\circ)$

22.3 ar tanh $r/\underline{\varphi} = x + j\,y$

Here again two equations provide solutions for x and y

$$\tanh 2x = \frac{2\,r\cos\varphi}{1 + r^2} = \frac{2\cos\varphi}{1/r + r}$$

$$\tan 2y = \frac{2\,r\sin\varphi}{1 - r^2} = \frac{2\sin\varphi}{1/r - r}$$

Example: $0.333/\underline{39.46°} = \tanh(x + j\,y)$ cf. Section 20.3.

$$\tanh 2x = \frac{2\cos 39.46°}{3.004 + 0.333}; \qquad 2x = 0.500, \quad x = 0.25$$

$$\tan 2y = \frac{2\sin 39.46°}{3.004 - 0.333}; \qquad 2y = 25.40, \quad y = 12.7°$$

$$0.333/\underline{39.46°} = \tanh(0.25 + j\,12.7°)$$

Reference to sections 14.2 and 19.3 shows that the evaluation of the fractions and the reading of the values 2 x and 2 y from scales Th and T offer no difficulties. To find 1/r, use either scales D and DI or the LogLog scales e^{+x} and e^{-x}, as scales of reciprocals.

22.4 arc sin $r/\underline{\varphi} = x + j\,y$

The solution in this case follows the procedure of section 22.1, with substitution of the trigonometrical sine value:

$$M^2 = (a + 1)^2 + b^2 \qquad \sin x = \frac{M - N}{2}$$

$$N^2 = (a - 1)^2 + b^2 \qquad \sinh y = \frac{b}{\cos x}$$

Example: $0.552/\underline{22.4°} = \sin(x + j\,y)$ cf. Section 21, ex. 1.

From $0.552/\underline{22.4°} = 0.51 + j\,0.21$,

$$M^2 = 1.51^2 + 0.21^2; \qquad M = 1.524$$

$$N^2 = 0.49^2 + 0.21^2; \qquad N = 0.532$$

$$\sin x = \frac{1.524 - 0.532}{2} = 0.496; \quad x = 0.52$$

$$\sinh y = \frac{0.21}{\cos 29.8°} = 0.242; \quad y = 0.24$$

22.5 arc cos $r/\underline{\varphi} = x + j\,y$

A further modification of the equations of section 22.1 is all that is required:

$$M^2 = (a + 1)^2 + b^2 \qquad \cos x = \frac{M - N}{2}$$

$$N^2 = (a - 1)^2 + b^2 \qquad \sinh y = \frac{b}{\sin x}$$

Example: $0.899/7.7° = \cos(x + j\,y)$ cf. section 21, ex. 2

$$M^2 = 1.891^2 + 0.1205^2; \qquad M = 1.895$$

$$N^2 = 0.109^2 + 0.1205^2; \qquad N = 0.162$$

$$\cos x = \frac{1.985 - 0.162}{2} = 0.867; \quad x = 0.52$$

$$\sinh y = \frac{0.1205}{\sin 29.8°} = 0.242; \quad y = 0.24$$

22.6 arc tan $r/\varphi = x + j\,y$

The required formulae are:

$$\tan 2x = \frac{2\,r\cos\varphi}{1 - r^2} = \frac{2\cos\varphi}{1/r - r}$$

$$\tanh 2y = \frac{2\,r\sin}{1 + r^2} = \frac{2\sin}{1/r + r}$$

23. Examples of application in practice

Several typical examples will be discussed to demonstrate the usefulness of the ARISTO HyperLog and ARISTO HyperboLog in electrical engineering calculations. In view of the fact that various terminologies are used in electrical engineering, the symbols employed in this text are listed below.
Symbols used in the complex representation of the Four Terminal Network and Transmission Line Theory:

Z	Impedance	$\mathbf{Z}_{oc}$	Open Circuit Impedance
Y	Admittance	$\mathbf{Z}_{sc}$	Short Circuit Impedance
$\mathbf{Z}_0$	Characteristic Impedance		
g	$= \gamma \times l = a + j\,b$ = Propagation Factor (Propagation Constant × length)		
a	Attenuation Factor (Attenuation Constant × length)		
b	Phase Factor (Phase Constant × length)		
p	Reflection Coefficient	α	Attenuation Constant
γ	Propagation Constant	β	Phase Constant

Insertion of an attenuation mesh

It is required to insert an attenuation network with the attenuation factor 1.5 N (13.03 d B) between a generator with an internal resistance of 60 ohms and a load resistance of 60 ohms. Find the line resistance r_1 and the shunt resistance r_2 in a T-type mesh,

The following equations of the net work theory are employed:

(1) $\mathbf{V}_1 = \mathbf{V}_2 \cosh \mathbf{g} + \mathbf{Z}_0\,\mathbf{J}_2 \sinh \mathbf{g}$

(2) $\mathbf{Z}_0\,\mathbf{J}_1 = \mathbf{V}_2 \sinh \mathbf{g} + \mathbf{Z}_0\,\mathbf{J}_2 \cosh \mathbf{g}$

The problem as presented prescribes:

$\mathbf{g} = a + j\,b = 1.5 + j\,0.$

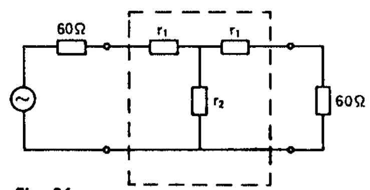

Fig. 86

With the receiving end open, equation (1) gives the voltage ratio:

(3) $\dfrac{\mathbf{V}_2}{\mathbf{V}_{2oc}} = \cosh \mathbf{g} = \cosh 1.5 = 2.352$

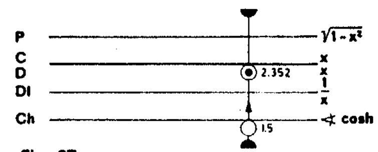

Fig. 87

Then, dividing equation (1) by equation (2) we obtain the open circuit input impedance $\mathbf{Z}_{oc}$, viz.:

$$\frac{\mathbf{Z}_{oc}}{\mathbf{Z}_0} = \coth \mathbf{g} = \coth 1.5 = 1.105$$

Set the cursor hairline to the value 1.5 on scale Th and find the result 1.105 below this value on scale DI.

The open circuit voltage ratio can be determined from the T-network:

$$\frac{\mathbf{V}_1}{\mathbf{V}_{2oc}} = \frac{r_1 + r_2}{r_2} = 2.352$$

See equation (3)

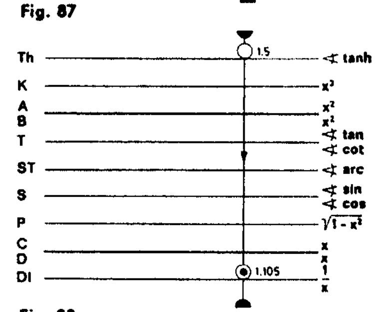

Fig. 88

Since $\mathbf{Z}_{oc} = \mathbf{Z}_0 \times \coth \mathbf{g} = 60 \times 1.105 = 66.3$ ohms

and $\mathbf{Z}_{oc} = r_1 + r_2 = 66.3$ ohms

it follows that $r_2 = \frac{66.3}{2.352} = 28.2$ ohms $\quad r_1 = 66.3 - 28.2 = 38.1$ ohms

Open circuit impedance and short circuit impedance of a copper transmission cable 0.2 mm in dia. and 10 km in length.

Let $\mathbf{Z}_0 = Z_0/\varphi$, where $Z_0 = 670$ ohms

$$\varphi = -41.6°$$

$$a = 0.814 \text{ N}$$
$$b = 0.843 \text{ rad} = 48.3°$$
$$\mathbf{g} = a + j\,b = 0.814 + j\,48.3°$$

Determination of open circuit impedance:

$$\mathbf{Z}_{oc} = \mathbf{Z}_0 \coth \mathbf{g} = 670\underline{/-41.6°} \times \coth(0.814 + j\,48.3°)$$

$$= 670\underline{/-41.6} \times \frac{\cosh(0.814 + j\,48.3°)}{\sinh(0.814 + j\,48.3°)}$$

$$= 670\underline{/-41.6} \times \frac{1.125\underline{/37°}}{1.173\underline{/59.1°}} = 642\underline{/-63.7}$$

$$\mathbf{Z}_{oc} = (284.2 - j\,575) \text{ ohms}$$

Determination of short circuit impedance:

$$\mathbf{Z}_{sc} = 670\underline{/-41.6°} \times \tanh(0.814 + j\,48.3)$$

$$= 670\underline{/-41.6°} \times \frac{1.173\underline{/59.1°}}{1.125\underline{/37°}} = 699\underline{/-19.5°}$$

$$\mathbf{Z}_{sc} = (659 - j\,233) \text{ ohms}$$

Complex voltage ratio of low pass filter

The problem consists of finding the complex voltage ratio of a low pass filter consisting of 6 similar T-sections with series resistance $R = 10\,k\Omega$ and shunt capacitances $C = 1\,\mu F$ operating with a frequency of 50 cycles. The complex voltage ratio of a symmetrical four terminal network with the receiving end open is:

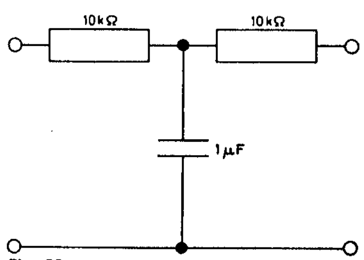

Fig. 89

$$\frac{\mathbf{V}_1}{\mathbf{V}_{2oc}} = \cosh \mathbf{g}$$

This equation is used to find the value of **g** for one section.

With $R = 10^4\ \Omega$ and

$$Y = \omega \times C = 2\pi \times 50 \times 10^{-6} \text{ find}$$
$$R \times Y = 3.142$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_{2oc}} = 1 + j\,R\,Y = 1 + j\,3.142 = \cosh \mathbf{g}$$

$$\text{arc}\cosh(1 + j\,3.142) = \mathbf{g} = x + j\,y$$

$1 + j\,3.142 = a + j\,b$

$1 + a = 2 \quad b = 3.142 \quad M = 3.725$ See chap. 19.2

$1 - a = 0 \quad b = 3.142 \quad N = 3.142$

$M - N = 0.583$

$\cos y = {}^1/_2\,(M - N) = 0.2915 \qquad y = 73.05°$

$$\sinh x = \frac{b}{\sin y} = \frac{3.142}{\sin 73.05°} = 3.285 \qquad x = 1.905$$

$$\mathbf{g} = 1.905 + j\,73.05°$$

Hence, for the 6 sections by simple addition of the transmission factors

$$6\,\mathbf{g} = 11.43 + j\,438.3° = 11.43 + j\,78.3°$$

The voltage measured at the terminals of the sixth T-section, is denoted by $\mathbf{V}_{7co}$ and the required relation is:

$$\frac{\mathbf{V}_1}{\mathbf{V}_{7oc}} = \cosh 6\,\mathbf{g} = \cosh(11.43 + j\,78.3) = r/\varphi$$

$$\tan\varphi = \tanh x \times \tan y = 1 \times \tan y$$
$$\varphi = y = 78.3°$$

$$r = \frac{\sinh x \times \sin y}{\sin\varphi} = \sinh x \times 1$$

$$r = \sinh 11.43 \approx \frac{1}{2}\,e^{11.43} = \frac{1}{2}\,(e^{5.175})^2$$

$$= \frac{1}{2}\,305^2 = 4.66 \times 10^4$$

$$\frac{\mathbf{V}_1}{\mathbf{V}_{7oc}} = 46600\underline{/78.3°}$$

Measuring the input impedance of a sending or receiving antenna.

Let us assume that a slotted line with a characteristic impedance of $\mathbf{Z}_0 = 60$ ohms be connected with a transmission line of the same characteristic impedance.

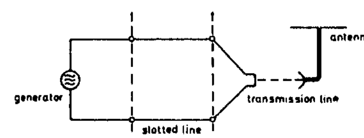

Fig. 90

The antenna at the end of the transmission line has an input impedance whose magnitude and phase is to be measured at one point as a function of frequency, A frequency of 100 mc corresponds to a wave length of 3 meters.

In order to eliminate the influence of the transmission line its terminals are short-circuited at the end. Now measure the distance l_1 between the voltage minimum and the end of the slotted line and likewise the values of the voltage minimum and voltage maximum.

The ratio between these two voltages is $m_1 = \frac{\mathbf{V}_{min}}{\mathbf{V}_{max}}$ (the reciprocal of the VSWR).

After reopening the circuit and reconnecting the antenna a second measurement will produce the new values l_2 and m_2.

Let $l_1 = 40$ cm $\quad m_1 = 0.15$ (short-circuited)

$l_2 = 70$ cm $\quad m_2 = 0.70$

The fundamental equations used in transmission line theory give us the reflection coefficient

$$\mathbf{p} = \frac{\mathbf{Z} - \mathbf{Z}_0}{\mathbf{Z} + \mathbf{Z}_0} = p/\varphi$$

At an arbitrary point of the lossless line, at distance x from the end, the impedance, which could be obtained by measurement if the left end were cut off, can be expressed by the formula

$$\frac{\mathbf{Z}_x}{\mathbf{Z}_0} = \frac{\mathbf{V}_x}{\mathbf{Z}_0\,\mathbf{J}_x} = \frac{e^{j\beta x} + \mathbf{p} \times e^{-j\beta x}}{e^{j\beta x} - \mathbf{p} \times e^{-j\beta x}}$$

At the point of minimum voltage V_{min} the expression $\mathbf{p} \times e^{-j\beta x}$ lags in phase by exactly 180° relative to $e^{j\beta x}$, so that in this particular case we can write

$$\frac{\mathbf{Z}_{x\,min}}{\mathbf{Z}_0} = \frac{1 + p\,/\underline{180^\circ}}{1 - p\,/\underline{180^\circ}} = \frac{1 - p}{1 + p}$$

On the other hand, the reciprocal of the voltage standing wave ratio (VSWR) is:

$$m = \frac{V_{min}}{V_{max}} = \frac{1 - p}{1 + p}$$

These voltages can be measured rather easily and their ratio then leads to the value $\mathbf{Z}_{x\,min}$ which must be retained for subsequent calculations.

The evaluation of the problem will be considerably simplified by assuming the input impedance $\mathbf{Z}$ to be caused by a loss-sustaining transmission line, short circuited at the end. This gives:

$$\frac{\mathbf{Z}}{\mathbf{Z}_0} = \tanh(a + j\,b)$$

wherein $\mathbf{g} = a + j\,b$ is the propagation factor of this imaginary line which must obviously have the characteristic impedance $\mathbf{Z}_0$. Now add the propagation factors of the transmission line with the same impedance $\mathbf{Z}_0$ and that of the section of the slotted line up to the voltage minimum.
For the sum of these lines then:

$$\frac{\mathbf{Z}_{x\,min}}{\mathbf{Z}_0} = \tanh(a + j\,b + a' + j\,b' + j\,b'')$$

where $a' + j\,b'$ represents the propagation factor of the transmission line and $j\,b''$ that of the slotted line section up to the minimum, assuming the same to be lossless. At the point of voltage minimum the impedance of the entire system has no reactive component. This simplifies the determination of the propagation factor.

(1) $m_1 = \tanh(a' + j\,b' + j\,b_1'') = 0.15$

(2) $m_2 = \tanh(a + j\,b + a' + j\,b' + j\,b_2'') = 0.70$

To have a simplified notation set:

$a' + j\,b' + j\,b_1'' = \mathbf{g}_1 \qquad b_2'' - b_1'' = \delta$

(1′) $m_1 = \tanh\,\mathbf{g}_1 = 0.15$

(2′) $m_2 = \tanh(\mathbf{g}_1 + a + j\,b + j\,\delta) = 0.70$

Since $\mathbf{g}_1$ is a real number which can be read on scale Th of the slide rule when the cursor is aligned to m = 0.15 on D, we obtain: $\mathbf{g}_1 = 0.1512$.

Similarly, by equation (2′) $\mathbf{g}_1 + a + j\,b + j\,\delta = 0.867$

$$a + j\,b + j\,\delta = 0.867 - 0.1512 = 0.7158$$

This equation gives $a = 0.716$, $b = -\delta$. The value δ is found by shifting the minimum point from the first to the second measurement. The propagation constant γ of the lossless slotted line is a pure phase constant, because in this case the attenuation constant α is zero. Hence $\gamma = j\,\beta$ and the propagation factor of the slotted line section between the minima of the two measurements is:

$$j\,\delta = j\,\beta\,(l_2 - l_1).$$

Generally: $\beta\,\lambda = 2\,\pi$ and thus

$$j\,\delta = j\,2\,\pi \times \frac{l_2 - l_1}{\lambda} = j\,2\,\pi\,\frac{70 - 40}{300}$$

$$j\,\delta = j\,2\,\pi \times 0.1$$

or, in degrees: $j\,\delta = j\,360^\circ \times 0.1 = j\,36^\circ$.

We now have the complete propagation factor of the imaginary line:

$$\mathbf{g} = a + j\,b = 0.716 - j\,36^\circ$$

The final operation now consists in computing the complex value of $\mathbf{Z}$ with the aid of the equation

$$\frac{\mathbf{Z}}{\mathbf{Z}_0} = \tanh(a + j\,b) = \tanh(0.716 - j\,36^\circ)$$

Chapter 20.3 shows how this expression is calculated.

$$\frac{\mathbf{Z}}{\mathbf{Z}_0} = \tanh(0.716 - j\,36^\circ) = \frac{0.977/\underline{-49.8^\circ}}{1.125/\underline{-24.05^\circ}} = 0.868/\underline{-25.8^\circ}$$

Then the final answer to our problem is the impedance:

$$\mathbf{Z} = 60 \times 0.868/\underline{-25.8^\circ} = 52.1/\underline{-25.8^\circ} = (47 - j\,22.6)\ \text{ohms}$$

24. The cursor and its marks

24.1 The mark 36
(Models 868, 0968, 869 and 0969 only)

The cursor has, on the front face (fig. 91 resp. 93) a short line upper right, corresponding to the value 36 on scales CF/DF, with respect to a value set on C/D under the middle cursor line. This enables multiplication by 36 to be performed by cursor transfer from C/D to CF/DF, a convenience when converting:

1 hour = 3600 seconds
1 m/s = 3.6 km/h
1° = 3600′′
100% = 360°
1 year = 360 days
1 kWh = 3.6×10^6 J
$\varkappa_{Al} = 36 \dfrac{m}{\Omega\ mm^2}$ (conductivity)

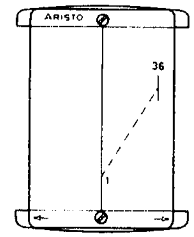

Fig. 91

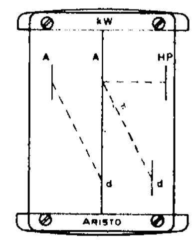

Fig. 92

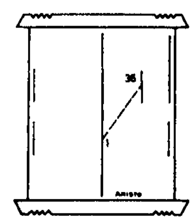

Fig. 93

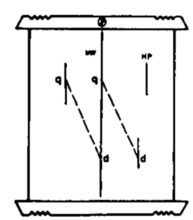

Fig. 94

24.2 Circular areas, weights of bar steel

On the reverse face of the cursor (fig. 92 resp. 94) are two short lines, upper left and lower right, displaced from the main cursor line by a distance proportional to the value $\pi/4 = 0.785$, (referred to the scale of squares). These are used in finding circular areas from the formula $A = d^2\pi/4$. If the main cursor line is brought over the diameter on scale D, the area can be read on scale A under the upper left short line. The same relationship holds between the lower right and main cursor lines.

Where the metric system of measures is in use, these special cursor lines can be used to find the weight of bar steel, because the specific weight of mild steel is 7.85 g/cm³. If the bar diameter is set on D with the main cursor line, the weight of unit length is read at once under the upper left short line. The index 1 of tcale B is set to the reading under the upper left line and the cursor moved over he total length of bar stock, to find the total weight.

24.3 The marks kW and HP

The interval between the upper right line and the center line represents the factor for converting kW to HP, and vice versa, on scale A (see fig. 92 resp. 94).

Hence, when the center hairline is set to 20 kW, for example, on the scale of squares, then the upper right line indicates the equivalent in HP viz. 27.2. Inversely, when the short right line is set to 7 HP the center line will produce the equivalent 5.15 kW.

24.4 The gauge marks 2π at the cursor L 0972

In addition to the mark 36, the front face of cursor pattern L 0972 (fig. 93) has at left and right hand sides reference lines for the factor 2π. These lines lie over scales C/D, CF/DF and are interrupted lines, to avoid confusion with the principal hairline. The 2π marks are of especial importance in frequency calculations.

Multiplication by 2π is accomplished by bringing the right hand 2π mark over the factor involved and reading the product under the left hand 2π mark. The converse procedure achieves division by 2π.

Example 1:

Find the frequency f of an oscillator if angular frequency $\omega = 372$ Hz from the relationship $f = \omega/2\pi$.

Move cursor to bring the left hand 2π mark over 3—7—2 on D. Beneath the right hand 2π mark read f = 59.2 Hz.

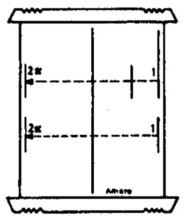

Fig. 95

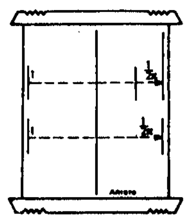

Fig. 96

Example 2:

Find the inductive resistance $X_L = 2\pi fL$ of a coil of frequency f = 59.2 Hz and inductance L = 21.5 mH.

Set 1 on CF under 5—9—2 on DF. Then move cursor so that the right hand 2π mark is over 2—1—5 on CF. The inductive resistance X_L = 8 Ω can then be read on DF, under the left hand 2π mark.

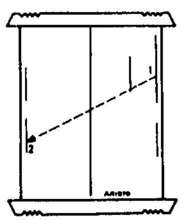

Fig. 97

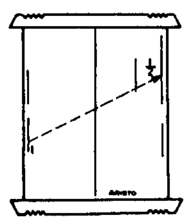

Fig. 98

In conjunction with the 2π marks reference to scale pairs C/D, CF/DF allows multiplication or division by 2 without slide setting. Multiplication by 2 follows when the upper right hand 2π mark is brought over the relevant factor on DF. On D beneath the left hand 2π mark the result is read at once. The converse sequence of cursor setting and reading provides the quotient of a division by 2.

25. The scale of preferred numbers 1364 (NZ scale)

25.1 Construction of the NZ scale

Standards and standardisation have become important factors in rationalised production and in this technology preferred numbers assume ever greater significance. Preferred numbers (BS 2045, ISO R 3, R 17) are selected values from a geometric series, developed from the denary number system. There is a very useful relationship between the graduations of the fundamental scale D and the associated mantissa scale L.

Opposite the equal intervals of the mantissa scale L are the corresponding plain numbers on scale D. The principal values tabulated as preferred numbers (BS 2045, ISO R 3, R 17) are these numbers, rounded off.

A scale of preferred numbers can be derived if the D scale is disregarded and the corresponding graduations of the mantissa scale are marked as preferred numbers.

On scale 1364, the ten numbered divisions of the upper mantissa scale are located over the preferred numbers of the R 10 series. The division of the mantissa scale into 20 equal divisions leads to the preferred numbers of the R 20 series and, with division into 40 equal intervals, to the R 40 series.

The preferred numbers are also marked against the mm scale: R 10 series by arrow points, R 20 by graduated lines and R 40 by dots.

25.2 Objects of the NZ scale

Scale 1364 is, first of all, an aid to memory, serving to exhibit instantly the commonly used preferred numbers. These are also of practical use when constructing single- and double-deck logarithmic graphs on normal squared graph paper. Because the multiplication or division of a preferred number by a preferred number always yields a preferred number, a graph doubly divided in preferred numbers serves for the graphic solution of problems.

The combination of preferred numbers and mantissas in a single scale offers the advantage that logarithmic approximations are simplified. The preferred numbers stand opposite the simplified logarithms of the mantissa scale and the latter can easily be added or subtracted mentally. By prefixing the characteristic, as must be done when using a table of logarithms, the decimal point can be correctly placed and the error in the result is a maximum of 3% if the R 40 series is used in the calculation.

In many cases the preferred number scale can equally well be used, if numbers are strongly rounded off. For example, if we take $\pi = 3.15$, for $\gamma = 7.85$ we take $\gamma = 8$. The mantissas corresponding to the preferred numbers are read from the mantissa scale set over the preferred numbers. It is very important to take into account the characteristic, on the presence of which this method of calculation essentially depends.

With complicated formulae, it is of advantage to write down the logarithms as read, so that a check can be made by addition. Natural numbers less than 1 (e. g., 0.8) are often best expressed as negative logarithms, e. g., $\lg 0.8 = -0.1$ is better put in the form $\lg 0.8 = 0.9 - 1$.

The graduations of L and D offer a more exact method of logarithmic calculation, since they provide, graphically, a three place table of logarithms.

25.3 Logarithmic scales

For the exact setting out of logarithmic scales or chart-nets, the scale 1364 carries logarithmically divided scales of base length 200 mm, 150 mm, 100 mm, 50 mm and 25 mm. Base lengths 125 mm and 250 mm can be taken from the slide scales of the rule.

25.4 Conversion factors for non-metric units

In the study of English, American and Continental literature, differences between anglo-saxon and metric units of measurement give rise to difficulty and relationship between the system must often be laboriously searched for in handbooks. This searching is obviated by the assembly of the most important conversion factors in a table on scale 1364.