New Statistical Scales
by Gregory J. McClure

This is a description of my new statistical scales available for Robert Wolf's excellent slide rule emulator. It is NOT an attempt to explain statistical methods or applications, but some examples will be included to show the use of the scales. I verified the scale readings by using tables from "CRC Standard Math Tables".

A sample slide rule with 8 scales can be found in the GJM subdirectory. It is called "gjm_8_sided_stat.txt". It can be modified for use with this tutorial.

Basic sets of statistical scales

The sets of scales are divided into two groups:

1) Log Gamma scales
2) Statistical distribution scales

Log Gamma scales

The Log Gamma scales represent the log (or ln) of the Gamma(x). X is located on the Gamma scale, and the log (or ln) of Gamma is found on the D scale (and the actual value, except for higher values of x, are found on the LLD [or LL] scales). For example, we want to determine the LogGamma of 10. Set the hairline to 10 on the LogGamma4 scale (the only one of the 6 LogGamma scales with 10 on it), and read 5.56 on the D scale (since this LogGamma scale is designed to read answers from 1 to 10). Also the LLD4 scale shows this to be about 360K (actual value is 362880). There are 6 LogGamma scales, ranging roughly from x=2.00054 thru 71. The last LogGamma scale does not have a corresponding LLD scale; the actual value of (Gamma(70) > 10^98, the LLD scales only go to 10^10). There are 5 LnGamma scales, ranging roughly from 2.0024 thru 38.3. Again the last LnGamma scale does not have a corresponding LL scale for actual value.

These scales were separated so that they can quickly be used (similar to LL and LLD scales). They only take one parameter, the height of the scale. Use the following lines in the slide rule definition file to describe these scales (do not use the … , it is there to show that there are multiple definitions):

- scale_LogGamma0 [height] … scale_LogGamma5 [height]
- scale_LogGamma0_down [height] … scale_LogGamma5_down [height]
- scale_LnGamma0 [height] … scale_LnGamma4 [height]
- scale_LnGamma0_down [height] … scale_LnGamma4_down [height]

Another example of scale use: suppose we want Log(Gamma(9.5)). If you are using the GJM 8 sided stat slide rule, move the hairline to 9.5 on the LG4 scale (found on side 8). The C scale reads approx. 5.08 (which is the Log(Gamma(9.5)). Gamma(9.5) can be found on the LLD4 scale, which reads around 120000. For comparison, Gamma(9) = 8! = 40320, and Gamma(10) = 9! = 362880.

Statistical distribution scales (general)

The statistical scales are all based on common statistical distributions. With the exception of the Chi Square Degrees of Freedom and Students-T Degrees of Freedom scales, all scale values represent the area (>=0.00001 and <=.99999) under the curve (a.k.a. the cumulative function) for the particular distribution (0 would mean no area exists for this x value, 1 would mean all the area under the curve). The value on the D scale represents the value of x equal to the area corresponding to the value on the distribution scale.

Normal distribution scales (and use of the decade parameter)

The simplest of the statistical distribution scales is the normal distribution scale. It has two flavors, the 1-tail and 2-tail. The distribution assumes center of 0 and standard deviation of 1. So at D scale = 1 (1 standard deviation, you will get a 1-tail value of .84 (= .5 + .34 for right side of distribution), or a 2-tail value of .68 (.34 on each side of the distribution).

These scales take two parameters, which decade to use (a floating value used to represents the power of 10 to multiply the D scale by for the x-value represented, decade -1.0 covers x = .1 to 1, decade 0.0 covers x = 1 to 10, decade 0.5 could be used to match the distribution to the DF10 scale covering x = sqrt(10) to 10*sqrt(10), etc) and the height of the scale (integer as usual). In this case, the scale parameter isn't of much use above 0.5. Use the following lines for normal distribution scale definitions:

- scale_NormalDist_1Tail [decade height]
- scale_NormalDist_1Tail_down [decade height]
- scale_NormalDist_2Tail [decade height]
- scale_NormalDist_2Tail_down [decade height]

As an example of use: You are told a population of normally distributed values has a mean of 3.3 and a standard deviation of 1.5. You want to know where 50% of the points will fall. We use the Nrm1 2t scale (side 2 of GJM 8-sided stat slide rule), and find that C = .6745. We can directly multiply this by 1.5 (the stdev) and read 1.012 on the D scale, then add 3.3 and get 4.312 for the high value. 3.3 – 1.012 = 2.288 for the low value. 50% of the points will fall between 2.288 and 4.312 for the described population.

Student's-T distribution scales

Closely associated with the normal distribution scale is the Student's-T distribution. The Student's-T distribution scale takes an extra parameter (degrees of freedom) in addition to the decade and height parameters. As the degrees of freedom go up, Student's-T more closely approaches the normal distribution (degrees of freedom > 30). Again both 1-tail and 2-tail versions are supplied. Example, for degrees of freedom = 2, hairline to .9 on the Normal Distribution scale as the desired area (confidence level), we find D-scale reads 1.886 if using the 1-tail scale, and 2.92 if using the 2-tail scale.

Interestingly, the degrees of freedom value is a floating value. Usually you will only use an integer value for this, but it must be formatted as a floating value (i.e. put a .0 after the integer) Use the following lines for Student's-T distribution scale definitions:

- scale_StudentsTDist_1Tail [decade degs_of_freedom height]
- scale_StudentsTDist_1Tail_down [decade degs_of_freedom height]
- scale_StudentsTDist_2Tail [decade degs_of_freedom height]
- scale_StudentsTDist_2Tail_down [decade degs_of_freedom height]

A couple of examples to show the use of Student's-T…

We have the following paired observations for two normal populations:

Dist 1 : 14.0, 17.5, 17.0, 17.5, 15.4
Dist 2 : 17.0, 20.7, 21.6, 20.9, 17.2

We want to test the hypothesis that the mean of the first sample is statistically the same as the mean of the second.

The differences are:

D : -3.0, -3.2, -4.6, -3.4, -1.8

Summing and dividing by the number of pairs we get Dbar = -3.20, Dstdev = 1.00

t for this sample is Dbar/Dstdev * sqrt(number of pairs) = -7.16

The degrees of freedom should be one less than the number of pairs, thus 4.

Using the t-Distribution scale with 4 degrees of freedom (modify the StudentsTDist_2Tail scales of gjm_8_sided_stat.txt if you wish to get this scale set, since they are setup for 2 degrees of freedom, you can just change that value to 4.0), we get .998 (confidence level) when x = 7.16. We are quite confident the population means are NOT statistically the same.

Another example, this time with unpaired values. We have two distributions we want to compare to see if they have the same mean (statistically).

Dist 1: 79, 84, 108, 114, 120, 103, 122, 120
Dist 2: 91, 103, 90, 113, 108, 87, 100, 80, 99, 54

The mean of the first set is 106.25, the mean of the second set is 92.5. We have 8 samples for set 1, and 10 samples for set 2, for a total of 18. Subtract 2 since we have 2 sets, and we have 16 degrees of freedom for this example.

Calculating the t-statistic is a bit more complicated when using unpaired sets of values… One divisor is the square root of the sum of the reciprocals of the number of points in each set, in this case sqrt(1/8 + 1/10). The second divisor is the square root of (sum of squares of all the values – for each set of values, number of points * mean squared, the resulting sum or difference divided by the deg_of_freedom). In our example, the sum of squares of all values is 180359, the number of points * mean squared for each set is 1411.133 for set 1, and 855.625 for set 2. So we are taking the sqrt { (180359 – 90312.5 – 85562.5) / deg_of_freedom } = 16.74. The numerator is simply the difference of the means (since we are checking to see if they are equal). Therefore

t = (mean1 – mean2) / { (sqrt[1/8 + 1/10]) * [16.74] } = 13.75 / 7.941 = 1.73.

Using the t-Distribution scale with 16 degrees of freedom, we get .897 (confidence level) when x = 1.73. We are somewhat confident the population means are not statistically the same. It barely fails the 90% confidence level test. If we were using 95% or 99% confidence level we would instead accept that they ARE statistically the same mean (that is, we are not confident enough to reject the hypothesis).

Chi Square distribution scales

Next is the Chi Square distribution. Again the Chi-Square distribution scale takes an extra parameter (degrees of freedom) in addition to the decade and height parameters. Only a 2-tail version is used (it is the only one I have ever seen tables for), and acts as the other distribution scales.

Actually, the Gamma (discussed next), Normal, and Chi Square distributions are all related to each other. Use the following for Chi Square distribution scale definitions:

- scale_ChiSquareDist [scale deg_of_freedom height]
- scale_ChiSquareDist_down [scale deg_of_freedom height]

Chi Square is used to decide if a given set of results matches what is expected. For our example, let's assume that O values represent the results observed, where E values represent the results expected…

O: 8, 50, 47, 56, 5, 14
E: 9.6, 46.75, 51.85, 54.4, 8.25, 9.15

Degrees of freedom is 6 – 1 or 5, X^2 = sum{ (O – E)^2 / E} = 4.84.

Using the Chi Square Distribution scale with 5 degrees of freedom, we get .564. We should accept that the distribution matches what is expected (since we are only 56.4% confident we can reject it, not enough for ANY confidence level used by statistics, which usually start at 90% to reject).

Degrees of freedom scales

Sometimes it is useful for a particular confidence level (area under the curve) to see what various degrees of freedom will yield that area. So a second Chi Square scale set and a second Student's-T scale set (2-tail only) were produced that takes the area as an extra parameter instead of the degrees of freedom, and yields the degrees of freedom on the scale itself. Use the following for these degrees of freedom scale definitions:

- scale_ChiSquareDegOfFreedom [decade area height]
- scale_ChiSquareDegOfFreedom_down [decade area height]
- scale_StudentsTDegOfFreedom [decade area height]
- scale_StudentsTDegOfFreedom_down [decade area height]

Using the Chi Square example above, if we are using 95% confidence level (use 5% scale for Chi Square, we are looking at the {100% - confidence level} for these scales), note that for 5 degrees of freedom we would have to have a X^2 value of over 11.07 (D scale) to reject the distributions as matching. So again, we are reasonably sure the observed distribution matches what is expected.

Gamma distribution scales

The Chi Square, Gamma, and Normal distribution scales are really special versions of the incomplete Gamma function. Gamma uses two values to shape the distribution, which I will call here the 'a' param for the left distribution shaper, and 'b' param for the right distribution shaper. The Gamma distribution has its own slide rule scale set. The scale definitions for the Gamma distribution are:

- scale_GammaDist [decade a_param b_param height]
- scale_GammaDist_down [decade a_param b_param height]

Two example scale sets of the Gamma Dist are found on gjm-8-sided-stat.txt side 1, a=1, b=2 and a=2, b=1. So, Gamma(1,2;.5) = .0902 (.5 on X, read Gamma Dist value on Gma1 1,2); whereas Gamma(2,1;.5) = .6321 (read Gamma Dist Value on Gamma1 2,1).

Example: let's say we know that major flooding occurs in a town every 6 years. Model when we would expect four additional floods to occur. We use Gamma(4,6) to model when we would expect the fourth flood to occur. Using the X scale, you read the number of years as Gamma(4,6) scale accumulates toward 1.

F and Beta distribution scales

The F distribution scales require two degree of freedom designations (numerator and denominator). It is related to the Beta distribution (which also is related to Student's-T for that matter). Both of these distributions are implemented by using the following scale definitions:

- scale_FDist [decade num_deg_of_freedom den_deg_of_freedom height]
- scale_FDist_down [decade num_deg_of_freedom den_deg_of_freedom height]
- scale_BetaDist [decade a_param b_param height]
- scale_BetaDist_down [decade a_param b_param height]

An example: let's say we randomly pick a sample of men and women from a population, the standard deviations of their weights are:

Men: Population 30, Sample of 12 men: 35
Women: Population 50, Sample of 7 women: 45

We want to know if we picked a sample with the same standard deviation (statistically).

The calculation for F is $(35^2/30^2) / (45^2/50^2) = 1.361 / 0.81 = 1.68$ {deg of freedom men / women = 11,6}
Or we could calculate: $(45^2/50^2) / (35^2/30^2) = 0.81 / 1.361 = 0.595$ {deg of freedom women / men = 6,11}

Setting up the F Dist scales you will find that F dist is 0.78 (men/women) or 0.22 (women/men). Since this does not go over 90% (or under 10%), we conclude there is no evidence to reject the sample, that is, they can be considered to have the same standard deviation as the population.

Conclusion

That about covers the new scales. There are NO degrees of freedom scales for the other distributions that have degrees of freedom parameters so far. If you look at an F distribution table, you can see why it doesn't work very well for F (and believe me I tried!).

Enjoy, Greg.