A summary of the essay:

Mirifici Logarithmorum Canonis
Reconstructio

An exploration of the mathematics of Napier's logarithm

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MIRIFICI

LOGARITHMORVM CANONIS CONSTRVCTIO;

Et eorum ad naturales ipſorum numeros habitudines;

VNÀ CVM

Appendice, de aliâ eâque præſtantiore Logarithmorum ſpecie condenda.

QVIBVS ACCESSERE

Propoſitiones ad triangula ſphærica faciliore calculo reſolvenda: *Unà cum Annotationibus aliquot doctiſſimi* D. HENRICI BRIGGII, *in eas & memoratam appendicem.*

Authore & Inventore *Ioanne Nepero*, Barone *Merchiſtonii*, &c. Scoto.

EDINBVRGI,
Excudebat ANDREAS HART.
ANNO DOMINI 1619.

RECONSTRUCTION OF NAPIER'S LOGARITHM TABLE
An exploration of the mathematics of Napier's Mirifici Logarithmorum Canonis *Constructio*

Abstract

The essay titled above explores the mathematical foundations of John Napier's groundbreaking work on logarithms, with particular emphasis on his seminal text, Mirifici Logarithmorum Canonis Constructio. Through this exploration, I aim to reconstruct Napier's table of logarithms and to shed light on the innovative techniques and concepts he employed in its creation. In addition, this reconstruction is situated within the broader historical context of mathematics in the sixteenth and seventeenth centuries.

The Descriptio

In 1614, the Scottish theologian and mathematician John Napier published the *Mirifici Logarithmorum Canonis Descriptio*, containing a logarithm table for trigonometric quantities. The table concerns remarkably good numerical approximations of the exact Napier logarithms (in this summary denoted by the capital letter *L*) of the Sine, Cosine, and Tangent (capital letters), of angles in the first quadrant of a circle with radius $R = 10^7$ units, where the arc of the circle is divided into 90 x 60 = 5,400 minutes. See Figure 1. The calculation method used by Napier led, after 20 years of laborious calculation, to a logarithm table for trigonometric quantities, of which Table 1 shows the first page.

Deg. 0 +|—

mi	Sines	Logarith	Differen.	Logarith:	Sines	
0	0	Infinite.	Infinite.	.0	1000000.0	60
1	291	8142567	8142568	.1	1000000.0	59
2	582	7449419	7449421	.2	999999.8	58
3	873	7043952	7043956	.4	999999.6	57
4	1164	6756275	6756274	.7	999999.3	56
5	1454	6533131	6533130	1.1	999998.9	55
6	1745	6350810	6350808	1.6	999998.6	54
7	2036	6196659	6196657	2.2	999998.0	53
8	2327	6063128	6063126	2.8	999997.4	52
9	2618	5945345	5945342	3.5	999996.7	51
10	2909	5839986	5839814	4.3	999995.9	50
11	3280	5744676	5744671	5.2	999995.0	49
12	3491	5657665	5657658	6.2	999994.0	48
13	3781	5577622	5577615	7.3	999992.8	47
14	4072	5503514	5503506	8.4	999991.7	46
15	4363	5434522	5434513	9.6	999990.5	45
16	4654	5369984	5369973	10.9	999989.2	44
17	4945	5309360	5309348	12.3	999987.8	43
18	5236	5252202	5252188	13.8	999986.3	42
19	5527	5198136	5198120	15.4	999984.7	41
20	5818	5146843	5146836	17.0	999983.1	40
21	6109	5098054	5098045	18.7	999981.3	39
22	6399	5051534	5051514	20.5	999979.5	38
23	6690	5007083	5007060	22.4	999977.6	37
24	6981	4964524	4964499	24.4	999975.6	36
25	7272	4923703	4923676	26.5	999973.6	35
26	7563	4884483	4884454	28.7	999971.4	34
27	7854	4846743	4846712	30.9	999969.2	33
28	8145	4810376	4810343	33.2	999966.8	32
29	8436	4775286	4775250	35.6	999964.4	31
30	8726	4741385	4741347	38.1	999961.9	30

Min.

Deg. 89

Deg. 0 +|—

mi	Sines	Logarith.	Differen.	Logarith.	Sines	
30	8726	4741385	4741347	38.1	999961.9	30
31	9017	4708596	4708555	40.7	999959.3	29
32	9308	4676848	4676805	43.4	999956.6	28
33	9599	4646077	4646031	46.1	999953.9	27
34	9890	4616225	4616176	48.9	999951.1	26
35	10181	4587239	4587187	51.8	999948.2	25
36	10472	4559069	4559014	54.8	999945.2	24
37	10763	4531671	4531613	57.9	999942.1	23
38	11054	4505004	4504943	61.1	999938.9	22
39	11344	4479030	4478965	64.4	999935.7	21
40	11635	4453713	4453645	67.7	999932.3	20
41	11926	4429022	4428950	71.1	999928.9	19
42	12217	4404925	4404850	74.6	999925.4	18
43	12508	4381396	4381318	78.2	999921.8	17
44	12799	4358408	4358326	81.9	999918.1	16
45	13090	4335936	4335850	85.7	999914.3	15
46	13380	4313958	4313868	89.6	999910.5	14
47	13671	4292453	4292360	93.5	999906.5	13
48	13962	4271401	4271304	97.5	999902.5	12
49	14253	4250783	4250682	101.6	999898.4	11
50	14544	4230583	4230477	105.8	999894.2	10
51	14835	4210781	4210671	110.1	999890.0	9
52	15126	4191364	4191250	114.5	999885.6	8
53	15416	4172317	4172198	118.9	999881.1	7
54	15707	4153627	4153504	123.4	999876.6	6
55	15998	4135279	4135151	128.0	999872.0	5
56	16289	4117263	4117130	132.7	999867.3	4
57	16580	4100664	4100527	137.5	999862.5	3
58	16871	4082175	4082032	142.4	999857.7	2
59	17162	4065082	4064935	147.3	999852.7	1
60	17452	4048276	4048124	152.3	999847.7	0

Min.

Deg. 89

Table 1. First page of Napier's logarithm table from 1614. In the left column, the angles run from 0°0' to 0°60' = 1° from top to bottom. In the right column, the complementary angles run from 89°0' to 89°60' = 90° from bottom to top. Therefore, you read the left Sine column from top to bottom; the right Sine column from bottom to top.

The Constructio

In 1619, his son *Robert Napier* and the London mathematician Henry Briggs published the *Mirifici Logarithmorum Canonis Constructio* posthumously, with notes in which Napier explained the rather complex mathematical background of his calculation method. Napier had compiled this detailed description several years before the publication of the *Descriptio*, but never published it. See [1], p. xvi.

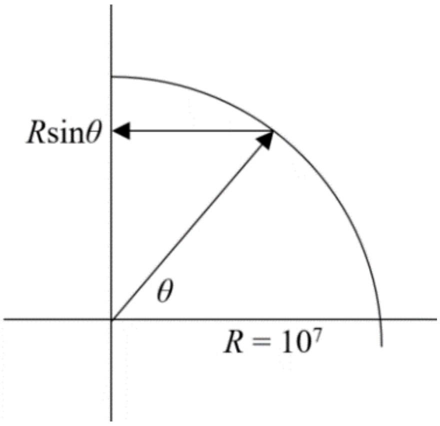


Fig. 1. Napier's quarter circle. In Napier's time, the Sine (capital letter) of an angle was understood to mean the modern sine (number between 0 and 1) multiplied by the radius R of the circle that was relevant at the time. Around 1600, trigonometric quantities were therefore understood not so much as ratios, but as lengths. Napier chooses the radius R equal to 10^7 units so that his Sines can have seven significant digits before the decimal point, in order to guarantee sufficient accuracy. See [1], p. 8.

Who was John Napier?

Napier, *Ioannes Neper*, was born in 1550 in Merchiston Castle, Edinburgh. The Napiers were among the oldest Scottish nobility; John's family tree goes back to 1068. See [11], p. 5 et seq., and see Figures 2 and 3. John was born during the Reformation in Scotland, where a fierce struggle was raging between Protestants and Catholics at the time. His family had become Protestant and John was very religious.

Fig. 2. John Napier (1550 – 1617), Lord of Merchiston near Edinburgh, Scotland. Portrait dated 1616; presented to the University of Edinburgh by his great granddaughter Margaret, who became Baroness Napier in 1686. Artist unknown. Source Wikipedia.org.

In 1563 he began studying theology at St. Andrews University, but for unclear reasons, he abandoned it after only a year.

In 1564 he embarked on a journey across mainland Europe—adventurous and dangerous for the time. It is unknown exactly what he was looking for there, but it is suspected that he studied theology in Geneva, among other places. See [11], p. 18. In 1571 he returned to Scotland, married, and became a landowner, the *8th Laird of Merchiston*, near Edinburgh.

In 1593 he published *The Plain Discovery of the Whole Revelation of Saint John*. An important conclusion he drew was: “The Pope is the Antichrist”. The book became a great success in Protestant Europe. Napier considered that his most important work, and not his discovery and construction of the logarithm. Mathematics seems to have been little more than an interesting hobby for Napier.

John Napier died in 1617, before the publication of the *Rabdologiae*, the book about his calculating rods. The *Constructio* was also published posthumously in 1619, with the collaboration of Briggs, among others.

MacDonald and Havil

In my essay of which this article is a summary I reconstruct the logarithm table in the *Descriptio* based on the still very readable English version of the *Constructio*, *The Construction of the Wonderful Canon of Logarithms*, from 1889, a translation by William Rae MacDonald [1].

Particularly noteworthy in this regard is the biography *John Napier, Life, Logarithms and Legacy*, by Julian Havil. See Havil (2014), [2]. This is not only a historical biography, but Napier's mathematical work is also discussed extensively and expertly by Havil. Havil also relies on the work of the mentioned MacDonald.

Fig. 3. Merchiston Castle in 1829.
From: https://thecorbettsociety.org.uk/the-1911-census/merchiston-road/

A mathematics book without formulas

When John Napier composed his *Constructio* at the beginning of the seventeenth century, neither analytical geometry nor infinitesimal calculus was yet available to articulate his profound mathematical insights concerning *continuously* varying quantities. Indeed, René Descartes' *La Géométrie* did not appear until 1637, while differential and integral calculus—also referred to as fluxional or infinitesimal calculus—originated in a seminal article by Gottfried Wilhelm Leibniz published in *Acta Eruditorum* in 1684, several decades after Napier's work. Likewise, Isaac Newton's *Philosophiæ Naturalis Principia Mathematica* did not appear until 1687.

A reader who consults John Napier's original *Constructio*, as well as William Rae Macdonald's English translation of 1889, encounters—entirely in keeping with early modern mathematical practice—a treatise devoid of symbolic notation. Although the text contains several diagrams featuring lines that might, from a contemporary perspective, be interpreted as number lines or coordinate axes, such an interpretation would be anachronistic. For Napier, these lines serve as representations of line segments undergoing *continuous* variation in length, rather than as loci of numerical values. The very notion of a coordinate system had yet to be formulated. Accordingly, Napier's mathematics is fundamentally Euclidean in its orientation and method. His reasoning is articulated almost exclusively through verbal exposition, a characteristic that imposes a significant interpretative burden upon the modern scholar engaging with the *Constructio*.

In the opening paragraph of the *Constructio*, John Napier writes that it (i.e., the logarithmic table) is “picked out from numbers progressing in *continuous* proportion.” That such a construction was achieved in the absence of the later mathematical frameworks of continuity is a remarkable accomplishment.

In the present summary, I shall employ analytical tools that were developed long after Napier’s lifetime and with which he was not acquainted, but of whose underlying principles he may be said to have possessed an intuitive apprehension.

Various logarithm tables

The calculation of logarithms at the end of the sixteenth and the beginning of the seventeenth century constituted a veritable tour de force. John Napier was the first to publish a table of logarithms, in his *Mirifici Logarithmorum Canonis Descriptio* (1614), intended in particular for trigonometric computations frequently required in disciplines such as astronomy, navigation, and geodesy. See Table 2.

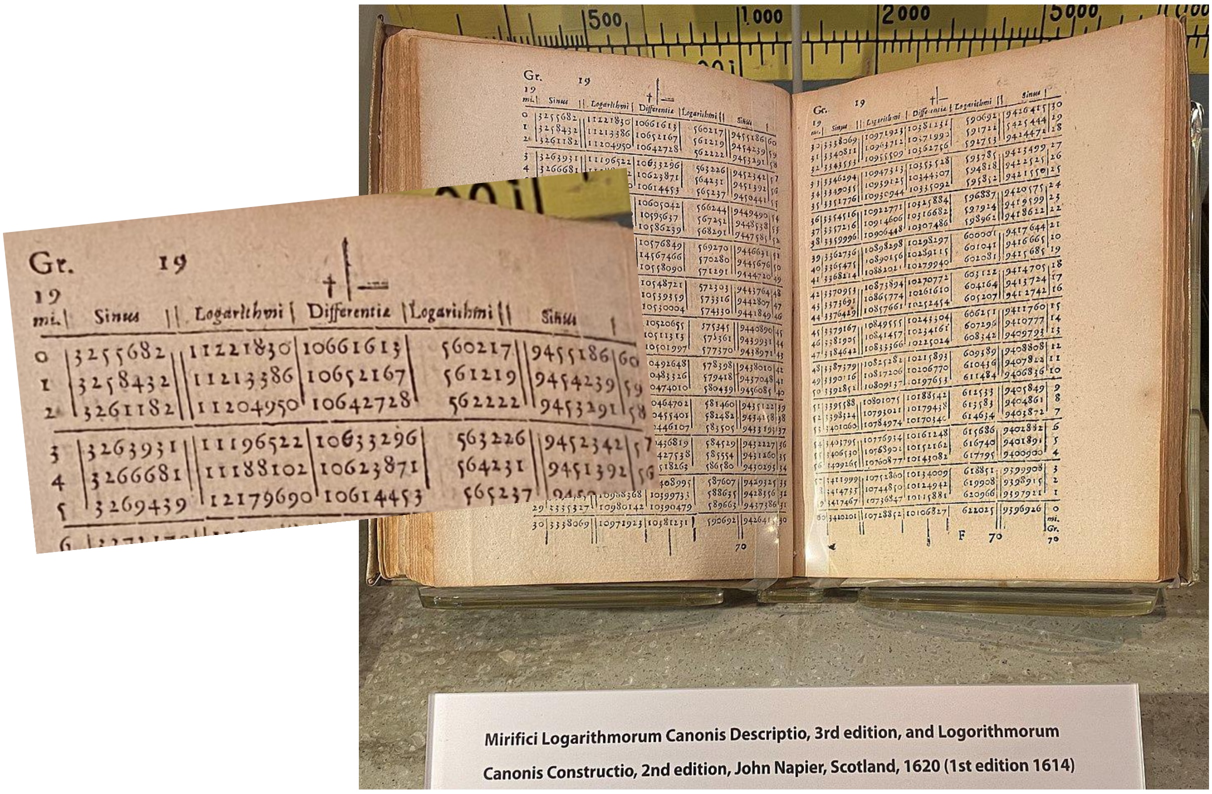


Table 2. Page for an angle of 19 degrees + 0 to 60 minutes from Napier's table.

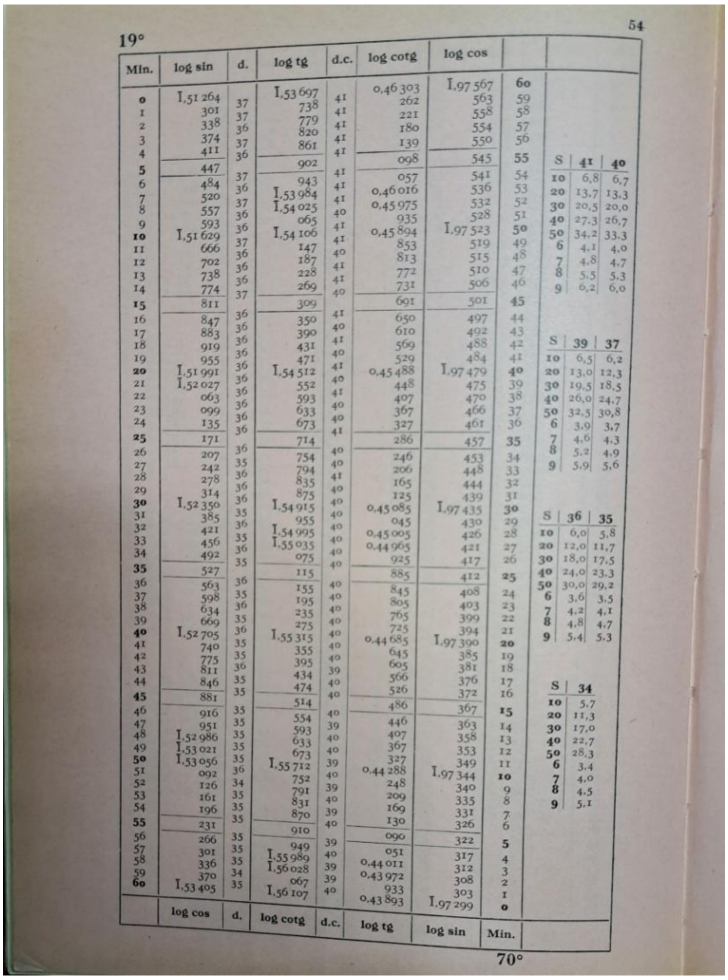
The London mathematician *Henry Briggs* (1561-1630) constructed his more generally applicable—though ultimately incomplete—table of logarithms on the basis of John Napier’s underlying conception of continuity, yet ultimately discontinued his efforts after more than a year of sustained computation (1617).

Ezekiel De Decker and Adriaan Vlacq

The Dutch scholars *Ezechiel de Decker* (1603/1604–1646/47), a surveyor and instructor in geometry and arithmetic, and *Adriaen Vlacq* (1600–1667), an astronomer, mathematician, and publisher, completed Henry Briggs's table after approximately ten years of work (1626/1627). See Van Poelje (2005) [7] and Van der Zijden (2000) [8].

In 1628, Vlacq published his *Arithmetica Logarithmica*, an international bestseller. Apart from the correction of computational errors, their table subsequently formed the basis for numerous logarithmic tables over the following 350 years. It is therefore noteworthy that this (complete) version has continued to be referred to as the Briggs logarithmic table. Although in the Netherlands (even during my school days in the 1960s), logarithm tables were sometimes informally called Briggs-Vlacq tables.

A reader consulting John Napier's table for the same angular value in degrees (19 degrees, as in Table 2) will observe that Napier's logarithmic values differ sometimes significantly from those found in later, more modern tables based on those of Ezechiel de Decker and Adriaen Vlacq (see Table 3). What, then, is the meaning of this discrepancy?



19° 54

Min.	log sin	d.	log tg	d.c.	log cotg	log cos	
0	Ī,51 264	37	Ī,53 697	41	0,46 303	Ī,97 567	60
1	301	37	738	41	262	563	59
2	338	36	779	41	221	558	58
3	374	37	820	41	180	554	57
4	411	36	861	41	139	550	56
5	447	37	902	41	098	545	55
6	484	36	943	41	057	541	54
7	520	37	Ī,53 984	41	0,46 016	536	53
8	557	36	Ī,54 025	40	0,45 975	532	52
9	593	36	065	41	935	528	51
10	Ī,51 629	37	Ī,54 106	41	0,45 894	Ī,97 523	50
11	666	36	147	40	853	519	49
12	702	36	187	41	813	515	48
13	738	36	228	41	772	510	47
14	774	37	269	40	731	506	46
15	811	36	309	41	691	501	45
16	847	36	350	40	650	497	44
17	883	36	390	41	610	492	43
18	919	36	431	40	569	488	42
19	955	36	471	41	529	484	41
20	Ī,51 991	36	Ī,54 512	40	0,45 488	Ī,97 479	40
21	Ī,52 027	36	552	41	448	475	39
22	063	36	593	40	407	470	38
23	099	36	633	40	367	466	37
24	135	36	673	41	327	461	36
25	171	36	714	40	286	457	35
26	207	35	754	40	246	453	34
27	242	36	794	41	206	448	33
28	278	36	835	40	165	444	32
29	314	36	875	40	125	439	31
30	Ī,52 350	35	Ī,54 915	40	0,45 085	Ī,97 435	30
31	385	36	955	40	045	430	29
32	421	35	Ī,54 995	40	0,45 005	426	28
33	456	36	Ī,55 035	40	0,44 965	421	27
34	492	35	075	40	925	417	26
35	527	36	115	40	885	412	25
36	563	35	155	40	845	408	24
37	598	36	195	40	805	403	23
38	634	35	235	40	765	399	22
39	669	36	275	40	725	394	21
40	Ī,52 705	35	Ī,55 315	40	0,44 685	Ī,97 390	20
41	740	35	355	40	645	385	19
42	775	36	395	39	605	381	18
43	811	35	434	40	566	376	17
44	846	35	474	40	526	372	16
45	881	35	514	40	486	367	15
46	916	35	554	39	446	363	14
47	951	35	593	40	407	358	13
48	Ī,52 986	35	633	40	367	353	12
49	Ī,53 021	35	673	39	327	349	11
50	Ī,53 056	36	Ī,55 712	40	0,44 288	Ī,97 344	10
51	092	34	752	39	248	340	9
52	126	35	791	40	209	335	8
53	161	35	831	39	169	331	7
54	196	35	870	40	130	326	6
55	231	35	910	39	090	322	5
56	266	35	949	40	051	317	4
57	301	35	Ī,55 989	39	0,44 011	312	3
58	336	34	Ī,56 028	39	0,43 972	308	2
59	370	35	067	40	933	303	1
60	Ī,53 405		Ī,56 107		0,43 893	Ī,97 299	0
	log cos	d.	log cotg	d.c.	log tg	log sin	Min.

S	41	40
10	6,8	6,7
20	13,7	13,3
30	20,5	20,0
40	27,3	26,7
50	34,2	33,3
6	4,1	4,0
7	4,8	4,7
8	5,5	5,3
9	6,2	6,0

S	39	37
10	6,5	6,2
20	13,0	12,3
30	19,5	18,5
40	26,0	24,7
50	32,5	30,8
6	3,9	3,7
7	4,6	4,3
8	5,2	4,9
9	5,9	5,6

S	36	35
10	6,0	5,8
20	12,0	11,7
30	18,0	17,5
40	24,0	23,3
50	30,0	29,2
6	3,6	3,5
7	4,2	4,1
8	4,8	4,7
9	5,4	5,3

S	34
10	5,7
20	11,3
30	17,0
40	22,7
50	28,3
6	3,4
7	4,0
8	4,5
9	5,1

70°

Table 3. Page for an angle of 19 degrees from a Noordhoff table.

A Neperian logarithm?

In some tables, the natural logarithm is called the *Neperian logarithm*. See, for example, the military tables book [3].

We also find this name, albeit less frequently, in electrical engineering. Electronics engineers working with transmission lines use the *Neper* as an alternative to the decibel: for two field quantities, the Briggs logarithm applies:

$$D = 20\log\frac{X_1}{X_2}\ \text{dB} \tag{1}$$

And with the natural logarithm:

$$N = \ln\frac{X_1}{X_2}\ \text{Np} \tag{2}$$

The potential attenuation of a signal in a telephone cable, for example, is a number of Np/km. See [4].

Compared to the dB, the Neper is hardly used anymore. This is remarkable because so many phenomena in electrical engineering are described using exponential functions. For example, the potential drop along a transmission cable as a function of length *L* is equal to:

$$V_L = V_{in} \exp\left(-\alpha L\right) \rightarrow \alpha L = \ln \frac{V_{in}}{V_L} \tag{3}$$

By expressing the attenuation coefficient *α* in Np/km, an additional computational step—required when using decibels—is avoided.

In this context, a connection may be observed between the natural logarithm and John Napier, although this is historically incidental rather than substantive. John Napier was one of the early discoverers of logarithms and is credited with introducing the term *logarithm* (from the Greek *logos* and *arithmos*, meaning *ratio number* or *proportion*). He also compiled the second logarithmic table—assuming that Jost Bürgi (1552–1632) produced the first—which was nonetheless the first to be published. However, Napier was not the inventor of the natural logarithm, which emerged only long after his death (1617).

In French usage, the natural logarithm is sometimes referred to as *logarithme népérien*, a convenient but historically misleading designation for the notation *ln*. Nevertheless, the term *Napierian logarithm* may suggest an intrinsic connection between Napier's logarithm and the natural logarithm, although such a connection is largely retrospective in nature.

The explosive base e

The natural logarithm (*logarithmus naturalis*) also has a base, namely *e* = 2.71828..., the transcendental base of all exponential growth, with ln *e* = 1. It is the base that unites the five basic constants of mathematics in Euler's equation, an equation that mathematicians consider the most beautiful expression:

$$e^{i\pi} + 1 = 0 \tag{4}$$

We can call e the *explosive base*: after all, Euler introduced it in a manuscript on cannons, the *Meditatio in Experimenta explosione tormentorum nuper instituta*, written in 1727/1728, more than 100 years after the death of John Napier.

We will see that the number *e* is indeed related to Napier's logarithm. Without realizing it, John Napier came very close to discovering *e*, but *e* is not the base of the Napierian logarithm!

A logarithm without the usual logarithm properties?

Napier's logarithm does not even have the most well-known properties that we attribute to logarithms. So, the definition rule, which we all learned in school,

$$a^c = b \leftrightarrow \log_a b = c\,, \tag{5}$$

does not apply to Napier's logarithm, as well as for example:

$$\log a + \log b = \log ab \tag{6}$$

But the rule of proportion that was of fundamental importance to Napier, namely

$$\frac{a}{b} = \frac{c}{d} \leftrightarrow \text{NapLog}\, a \text{ - NapLog}\, b = \text{NapLog}\, c \text{ - NapLog}\, d\,, \tag{7}$$

is indeed valid. Probably, Napier derived the term logarithm, *Logos Arithmos*, or ratio number, from this formula.

Napier's logarithm: two points in motion

Napier imagined two lines, one with a line segment OX, whose length

$$x = \text{Sin}\,\theta = R\sin\theta \tag{8}$$

becomes smaller and smaller as the angle θ decreases, and a second line with line segment OL, with a length equal to what we shall call the *exact Napier logarithm* of x, namely $L(x)$, which increases as x decreases. See Figure 4. The points X and L therefore move in opposite directions.

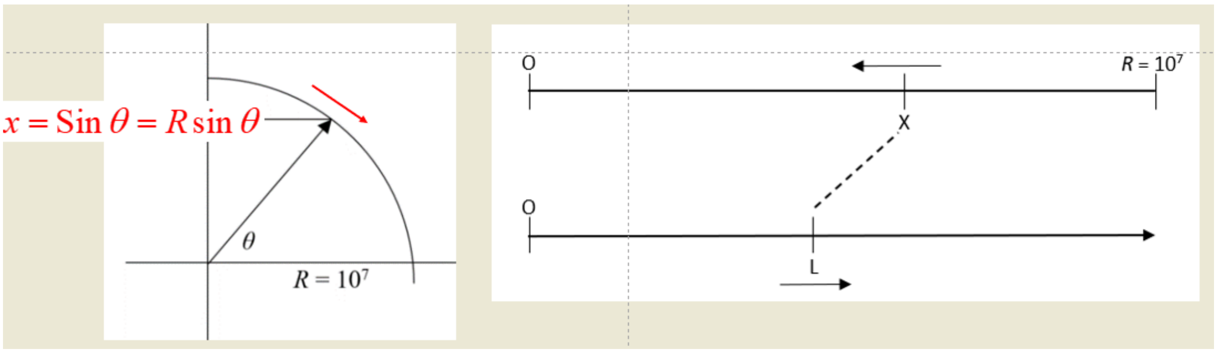


Fig. 4. Relationship between length x and the exact Napier logarithm L of x.

The continuous, exact, kinematic model yields logarithms per arc minute of $x = \text{Sin}\,\theta = R\sin\theta$, of angles $90°\,0' \geq \theta \geq 0°\,0'$ and radius $R = 10^7$.

Napier envisioned a continuous geometric progression. We would call it an exponential function. The English translation of the *Constructio* states: "it is picked out from numbers progressing in *continuous* proportion". See [1], p. 7.

For the following, see also Van der Salm (1999) [9] and Havil (2014) [2], pp. 96 - 130.

Napier devised the following:

1. There are two lines along which the points X and L move at different speeds.

2. On the upper line, X traverses the finite interval OR with length $R = 10^7$; on the lower line, L traverses a half-line to infinity.

3. X starts on the top line at $x = R = 10^7$. That starting point is the sine of 90°. Its initial velocity is $-R$ (m/s or a similar unit). X moves with decreasing, *negative* velocity, in the direction of O, for which $x = 0$.

4. As X moves to the left, the angles θ become smaller and smaller, and therefore the sines $x = \mathrm{Sin}\,\theta = R\sin\theta$ (= length of OX) also become smaller and smaller.

5. Point L, with distance L to O, traverses the bottom line, starting from O ($L = 0$), at a constant speed of $+R$ (m/s or similar unit), to $L = \infty$.

6. The further X lies to the left, the smaller x is, and the larger the corresponding exact Napier logarithm $L(x)$.

What is the relationship between X and L? Because X moves in the direction of O on the positive x-axis OR, its velocity is negative. Napier chose the speed of X equal to the negative distance $-x$. (We will omit the rather lengthy explanation of his motivation for this here.):

$$v_X = \frac{\mathrm{d}x}{\mathrm{d}t} = -x \tag{9}$$

At moment $t = 0$ is $v_X(0) = -R$ (m/s or similar unit).

From equation (9) and this initial value, we conclude:

$$x = R\left(\frac{1}{e}\right)^t \tag{10}$$

The distance x is apparently expressed (in our modern mathematics) as a power with base $base = \frac{1}{e}$, multiplied by $R = 10^7$.

Formula (10) shows what Napier must have meant by a continuous geometric sequence. (We would say an exponential function).

Formula (10) still contains the time t, which is not needed. We can eliminate it as follows. The relationship (10) between t and x is also:

$$t = \ln \frac{R}{x} \tag{11}$$

Simultaneously with point X, the point L starts at O on the bottom line with an equally large, but positive initial velocity $R = +10^7$ (m/s or similar unit). Unlike X, L moves at a *constant* instantaneous speed. At time t, point L has therefore covered the length:

$$L = R\,t \leftrightarrow t = \frac{L}{R} \tag{12}$$

Formulas for the Napier-logarithm

The formulas for the exact Napier logarithm are therefore:

$$\begin{cases} x(L) = R\left(\frac{1}{e}\right)^{L/R} \\ \mathrm{NapLog}(x) = L(x) = -R\ln\frac{R}{x} = R \cdot \log_{1/e}\left(\frac{x}{R}\right) \end{cases} \tag{13}$$

Calculating the Napier logarithms using formula (13) is easy with Excel. See the table page in Table 4. It is almost unbelievable that Napier produced this work with pen and on paper, without any calculating aid, while he did not have access to modern mathematical formulas.

Napier calculated the numerical approximations of the exact logarithms in a completely different way, namely by linking a geometric sequence and an arithmetic sequence. See also Donners, (2002) [5]. In my large essay, of which this article is a summary, Napier's method is explained in detail.

However, the fundamental idea of his logarithm construction is shown above. All this is very astonishing, as is evident from the following quote by Lord Moulton, during the conference on the 400th anniversary of Napier's Logarithm, Edinburgh 1914:

> ***No previous work had led up to it, nothing had foreshadowed it or heralded its arrival. It stands isolated, breaking upon human thought abruptly, without borrowing from the works of other intellects or following known lines of mathematical thought.***

From: Lynne Gladstone-Millar: *John Napier, Logarithm John,* (2013), [11], p. 42.

Degr	Min	Sin	Log	plus/min	Log	Sin			Min	Sin	Log	plus/min	Log	Sin	
19	0	3255682	11221835	10661617	560217	9455186	60		30	3338069	10971927	10381235	590692	9426415	30
	1	3258432	11213391	10652171	561220	9454238	59		31	3340810	10963717	10371994	591723	9425444	29
	2	3261182	11204954	10642732	562223	9453290	58		32	3343552	10955514	10362759	592755	9424471	28
	3	3263932	11196526	10633300	563227	9452341	57		33	3346293	10947318	10353531	593787	9423498	27
	4	3266681	11188106	10623875	564231	9451391	56		34	3349034	10939130	10344310	594820	9422525	26
	5	3269430	11179694	10614457	565237	9450441	55		35	3351775	10930950	10335095	595855	9421550	25
	6	3272179	11171290	10605046	566244	9449489	54		36	3354516	10922777	10325887	596890	9420575	24
	7	3274928	11162893	10595641	567252	9448537	53		37	3357256	10914612	10316685	597926	9419598	23
	8	3277676	11154505	10586244	568261	9447584	52		38	3359996	10906454	10307490	598964	9418621	22
	9	3280424	11146124	10576854	569270	9446630	51		39	3362735	10898303	10298301	600002	9417644	21
	10	3283172	11137751	10567471	570281	9445675	50		40	3365475	10890161	10289120	601041	9416665	20
	11	3285919	11129386	10558094	571292	9444720	49		41	3368214	10882025	10279944	602081	9415686	19
	12	3288666	11121029	10548724	572305	9443764	48		42	3370953	10873897	10270775	603122	9414705	18
	13	3291413	11112680	10539362	573318	9442807	47		43	3373691	10865777	10261613	604164	9413724	17
	14	3294160	11104339	10530006	574333	9441849	46		44	3376429	10857664	10252457	605207	9412743	16
	15	3296906	11096005	10520657	575348	9440890	45		45	3379167	10849558	10243307	606251	9411760	15
	16	3299653	11087679	10511315	576364	9439931	44		46	3381905	10841460	10234164	607296	9410777	14
	17	3302398	11079361	10501979	577382	9438971	43		47	3384642	10833369	10225027	608342	9409793	13
	18	3305144	11071051	10492651	578400	9438010	42		48	3387379	10825286	10215897	609389	9408808	12
	19	3307889	11062748	10483329	579419	9437048	41		49	3390116	10817210	10206773	610436	9407822	11
	20	3310634	11054453	10474014	580439	9436085	40		50	3392852	10809141	10197656	611485	9406835	10
	21	3313379	11046166	10464706	581460	9435122	39		51	3395589	10801080	10188545	612535	9405848	9
	22	3316123	11037887	10455404	582482	9434157	38		52	3398325	10793026	10179440	613585	9404860	8
	23	3318867	11029615	10446110	583505	9433192	37		53	3401060	10784979	10170342	614637	9403871	7
	24	3321611	11021351	10436822	584529	9432227	36		54	3403796	10776940	10161250	615689	9402881	6
	25	3324355	11013095	10427541	585554	9431260	35		55	3406531	10768908	10152165	616743	9401891	5
	26	3327098	11004846	10418266	586580	9430293	34		56	3409265	10760883	10143086	617797	9400899	4
	27	3329841	10996605	10408998	587606	9429324	33		57	3412000	10752865	10134013	618853	9399907	3
	28	3332584	10988371	10399737	588634	9428355	32		58	3414734	10744855	10124946	619909	9398914	2
	29	3335326	10980145	10390483	589663	9427386	31		59	3417468	10736852	10115886	620966	9397921	1
	30	3338069	10971927	10381235	590692	9426415	30		60	3420201	10728856	10106832	622025	9396926	0
							Min						**Degr**	**70**	**Min**

Table 4. Example table page with Napier logarithms for the angle of 19 degrees, generated with Excel.

Conclusions

- Napier was far ahead of his time with his mathematical insight of continuous motion
- Actually, Napier used analytical mathematics that was discovered more than a century later
- Napier used clever interpolations to actually calculate his logarithms
- Napier's logarithm is NOT the natural logarithm.
- Napier's logarithm is not a logarithm in the modern sense.
- Many of the usual rules of calculation are not applicable to the Napier logarithm
- Napier took some 20 years to complete the calculations.
- Besides, Napier Is the (re-)inventor of the decimal point and the modern decimal representation of fractions

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