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# Diagonal Scales - A Critical Assessment

## Introduction

During my Gunter rule investigations, I always passed quickly over the diagonal scales ("transversaalschaal" in Dutch) on the front, expecting no surprises in such an elementary drawing tool.
Until a collector friend asked me for an explanation of the diagonal scale, which I gave him to the best of my knowledge (see following section on usage).
To my surprise he answered that his own Gunter rule did not match this explanation, and after further browsing we established that a number of known Gunter rules appear to have their diagonals slanted -or even numbered- the wrong way around.
This article will address the intended use of the diagonal scale, and the discovery of presumed errors in the drawing of diagonal lines on some specimens.

## Usage of diagonal scales

Diagonal scales occur not only on Gunter rules, but also on many other rules or drawing instruments, like carpenter rules, scaling rules, plain scales and protractors.
These scales are used with a set of dividers to either construct or to measure length quantities as distances between the tips of the dividers. The dividers are used not only on the diagonal scale, but also to move a distance between diagonal scale and the real world, like a drawing, map or any physical object.
In its basic form the diagonal scale allows a distance to be measured in three decimals.

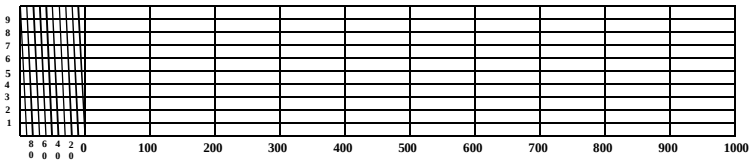



*Figure 1 - Basic diagonal scale*

Figure 1 shows a scale of 1000 units, with hundreds to the right of point zero and tens to the left. Each of the tens should in principle be divided again in 10 units (most significant digits), but space does not really allow this. Therefore the units within the tens are projected vertically upwards onto the respective horizontals, which are numbered 1 to 10. A diagonal is then drawn through all intersections of the vertical projection of a unit and the horizontal line bearing that unit's number.
This diagonal can now be used to determine the unit value of a distance: for example, unit 5 (of 25) in figure 2 projects along the diagonal onto horizontal no. 5, which is much easier to read due to the steep slope of the diagonal.

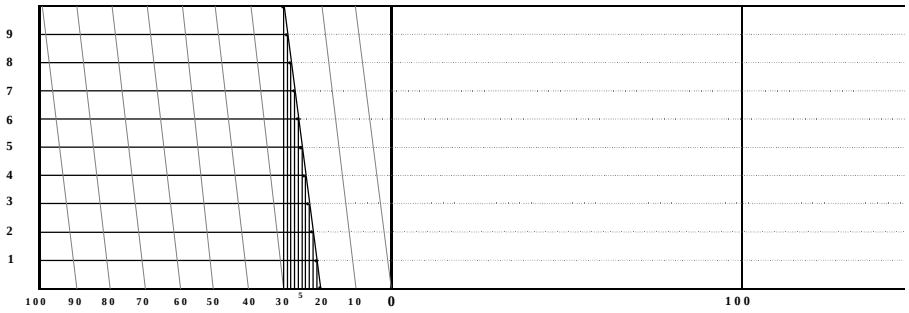
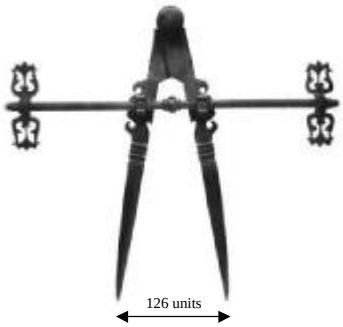



*Figure 2 - Diagonal as vertical projection of units*

## Example

Figure 3 shows the dividers hovering above a demonstration distance of 126 units, composed of the summation of one hundreds unit, two tens units, and six units as projected on the diagonal between 20 and 30 (see the distance between the two small circles).




For construction, the right tip of the dividers is put on 100 of horizontal 6 (right circle), the left tip is put on the intersection of the same horizontal and the diagonal between 20 and 30 (left circle). For measuring this distance of 126 units, the tips should be placed such that the left is in the diagonal area while the right tip is on a hundreds vertical. Then the dividers as a whole should be moved up or down tentatively until reaching a horizontal, where the left tip touches the intersection of that horizontal and a diagonal (left circle) while the right tip is on a hundreds vertical (right circle).

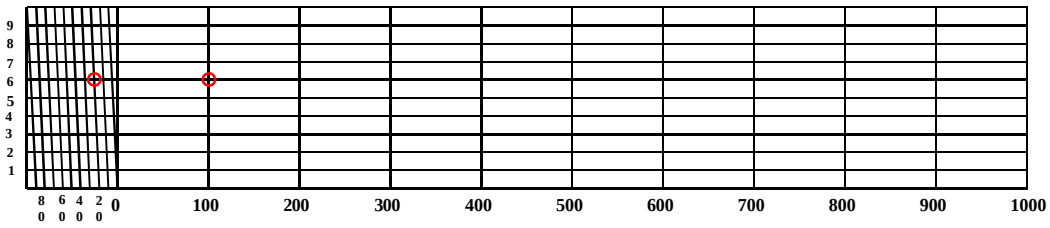



*Figure 3 - Diagonal scale in use*

This use of the dividers on the diagonal scale can be considered to be a calculation: the addition

1*100 + 2*10 + 6

In reference [4] this principle is extended, for educational purposes, to general addition on the diagonal scale.

## V-type diagonal scale

Another form of diagonal scale handles only two decimals of units, see fig. 4. The diagonals look like an upside-down "V". The advantage is that the height of the scale can be smaller, with only 6 horizontals, but at the cost of precision in number of decimals. In many cases the slope of V-type diagonals is so low that there is no real improvement in reading accuracy.

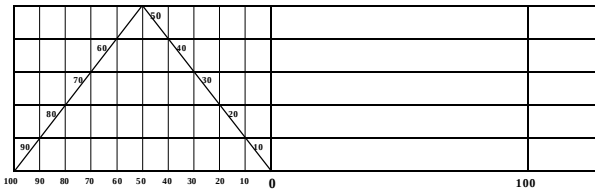



*Figure 4: V-type diagonal scale*

## Composite diagonal scale

When rules needed multiple diagonal scales, for different scaling factors, it was usual practice to combine two of them for reasons of space efficiency. Architect's diagonal scale rules have four differently scaled diagonal sets, two combined on each side. Gunter rules have two diagonal sets, combined in one scale, see figure 5: one for inches (top scale and right diagonals box), and one for half-inches (bottom scale and left diagonals box). Because the one-inch diagonals are at the right side, the inches themselves are numbered from right to left.

Note that, because of the methods described in the usage section, the diagonals have to slant out from point zero, resulting effectively in diagonals which have the same slanting direction in the left and the right box.

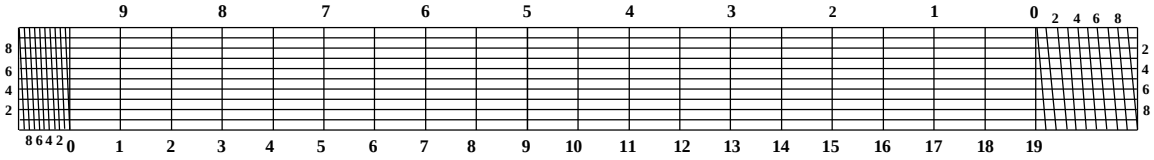



*Figure 5 - Correct drawing of double diagonal scale on most Gunter rules*

## Literature sources

Looking for literature sources on diagonal scales, it is disappointing that neither Bion [1] nor Hambly [2] give a satisfactory description of the usage and the resulting design of the double diagonal scale.

The best textual description I have found so far is in Robertson's '*A Treatise of Mathematical Instruments*' [3], see the following excerpt (next page):

**“Of the Lines of Equal Parts.**

*LINES of equal parts are of two sorts, viz. simply divided, and diagonally divided. Pl. V.*

*Simply divided* . . . . . (describes the single scale of a common rule)

*Diagonally divided. Draw eleven lines parallel to each other, and at equal distances; divide the upper of these lines into such a number of equal parts as the scale to be expressed is intended to contain; and from each of these divisions draw perpendiculars through the eleven parallels, (Fig. 15) subdivide the first of these divisions into 10 equal parts, both in the upper and lower lines; then each of these subdivisions may also be subdivided into 10 equal parts, by drawing diagonal lines; viz. from the 10th below, to the 9th above; from the 9th below to the 8th above; from the 8th below to the 7th above, &c. till from the 1st below to the 0th above, so that by these means one of the primary divisions on the scale, will be divided into 100 equal parts.*

*There are generally two diagonal scales laid on the same plane or face of the ruler, one being commonly half the other. (Fig. 15.)*

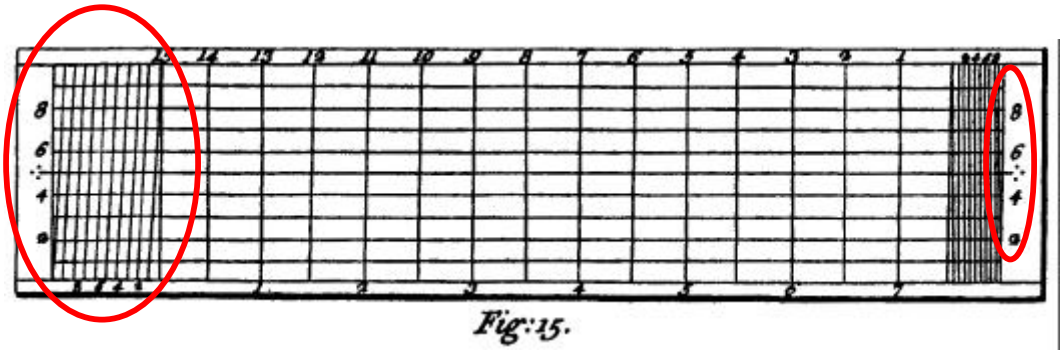


Fig: 15.

**WRONG!!** **WRONG!!**

*The use of the diagonal scale is much the same with the simple scale; all the difference is, that a plan may be laid down more accurately by it: because in this, a line may be taken of three denominations; whereas from the former, only two could be taken.*

*Now from this construction it is plain, if each of the primary divisions represent 1, each of the first subdivisions will express 1/10 of 1; and each of the second subdivisions, (which are taken on the diagonal lines, counting from the top downwards) will express 1/10 of the former subdivisions, or a 100th of the primary divisions; and if each of the primary divisions express 10, the each of the first subdivisions will express 1, and each of the 2d, 1/10; and if each of the primary divisions represent 100, then each of the first subdivisions will be 10; and each of the 2d will be 1, &c.*

*Therefore to lay down a line, whose length is express'd by 347, 34 7/10 or 3 47/100 whether leagues, miles, chain, &c.*

*On the diagonal line, joined to the 4th of the first subdivisions, count 7 downwards, reckoning the distance of each parallel 1; there set one point of the compasses, and extend the other, till it falls on the intersection of the third primary division with the same parallel in which the other foot rests, and the compasses will then be opened to express a line of 347, 34 7/10, or 3 47/100, &c. “*

## Errors in drawing the diagonals

When we look at Robertson's ‘Fig: 15’, the top scale presents half-inches, right to left, with the correct diagonals set at the right, but numbered in the wrong direction.
The full-inch scale on the bottom, left to right, should have diagonals in the left box, slanting bottom up to the left. But we observe that the diagonals in the left side box are slanted the wrong way around!

One could argue that the diagonals are to be read top to bottom, but then the numbering of the horizontals is wrong.
When we look at Bion's picture of a double diagonal scale [2], we see a comparable drawing error.
Looking at actual specimens of Gunter rules, some show the same type of error in the slanting direction of the diagonals, hardly surprising when major handbooks on the subject give the wrong direction.

As a specific example, the Gunter rule, drawn in [5] as an exact copy of an English-made "Potter Poultry" rule, also has the same type of error. Among the other erroneous samples encountered, there was even a diagonal scale with all diagonals drawn exactly vertical!
Looking at some architect's scaling rules with diagonals, no such errors have yet been detected.

## Conclusions

The relative large number of errors in diagonal scale drawings and actual Gunter rules would suggest that a complete understanding of the diagonal scale was not widespread, and that rule makers often copied errors, probably in good faith.
Maybe the inclusion of a diagonal scale was more an ornamental embellishment of the rule?
The investigations, leading to this article, have been restricted to literature and rule samples within my reach. I would appreciate very much if readers would check their own library and collection items, and report back to me other occurrences of diagonal drawing errors.

Any other comments on this article will be very welcome.


## Acknowledgments

Discussions with Sigismond Kmiecik and Leo de Haan have substantially contributed to this article.


## Literature

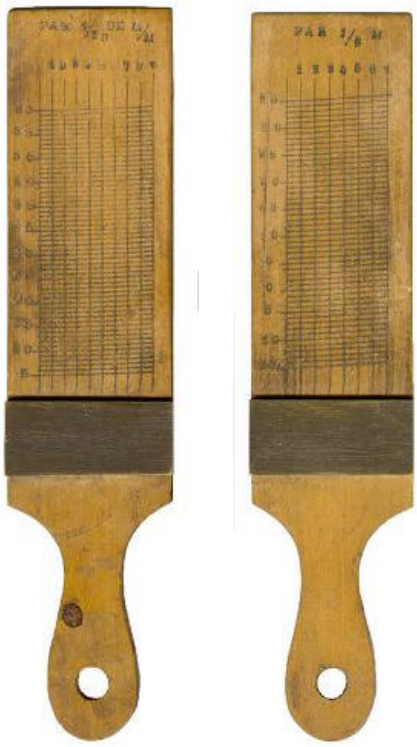

[1] M. Bion & E. Stone, '*Construction and principal uses of Mathematical Instruments*', 1758 - facsimile published by Astragal Press, 1995 (Ch. V and Plate IV, fig. 2)
[2] M. Hambly, '*Drawing Instruments*', 1988, Ars Mundi (Chapter 7 on scales)
[3] J. Robertson, '*A Treatise of Mathematical Instruments*', 1775 - facsimile published by The Invisible College Press, 2002 (Section VII), on-line at http://www.orbitals.com/books/tmi/excerpt.html#sect7
[4] J.L. Sieber, '*Calculating Using the Diagonal Scale*', Journal of the Oughtred Society, Vol. 10, No. 2, Fall 2001
[5] B.E. Babcock, '*Some Notes on the History and Use of Gunter's Scale*', Journal of the Oughtred Society, Vol. 3, No. 2, Fall 1994


# Andere transversaalschalen

De twee hierbij afgebeelde transversaalschaaltjes (ca. 2/3 ware grootte) zijn waarschijnlijk van Franse makelij. Beide bezitten een opgeklonken messing plaatje als aanslag (?).

De links afgebeelde is van verzilverd messing, en enkelzijdig belijnd met een schaal van 75 mm lengte. De 0-lijn ligt bij de aanslag, de transversalen beginnen bij 5. Met de punt van de steekpasser tegen de aanslag kan elke maat tussen 5 en 76 mm op 1/10 mm nauwkeurig worden afgemeten.

De rechter is van buxushout (palmhout) en dubbelzijdig belijnd.
Eén zijde (PAR 1/10 DE M/M) heeft een schaallengte van 60 mm en is vergelijkbaar met de vorige.
De andere zijde (PAR 1/8 MM) heeft een afwijkende schaal. De 0-lijn ligt 18 mm boven de aanslag. De maatstreepjes naast de transversalen zijn voorzien van de getallen 15 – 10 – 5 – 0 – 5 – 10 – enz. tot 55. Tussen elke transversaal is de afstand 1,1 mm.
De transversalen snijden 8 lijnen, wat betekent dat een afstand op 0,1375 mm nauwkeurig kan worden gemeten.
Het idee is dat de latjes mogelijk bestemd zijn voor een bepaald vakgebied met – zeker bij het tweede latje – een afwijkend maatsysteem.



**Wie kan meer zeggen over doel en gebruik van deze transversaalschaaltjes?** (Harrie)